39

PARTICLES BEHAVING AS WAVES

39.1. IDENTIFY and SET UP: $\lambda = \frac{h}{p} = \frac{h}{mv}$. For an electron, $m = 9.11\times10^{-31}$ kg. For a proton, $m = 1.67\times10^{-27}$ kg.

EXECUTE: (a) $\lambda = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(9.11\times10^{-31}\text{ kg})(4.70\times10^{6}\text{ m/s})} = 1.55\times10^{-10}\text{ m} = 0.155\text{ nm}$

(b) λ is proportional to $\frac{1}{m}$, so $\lambda_\text{p} = \lambda_\text{e}\left(\frac{m_\text{e}}{m_\text{p}}\right) = (1.55\times10^{-10}\text{ m})\left(\frac{9.11\times10^{-31}\text{ kg}}{1.67\times10^{-27}\text{ kg}}\right) = 8.46\times10^{-14}$ m.

EVALUATE: For the same speed the proton has a smaller de Broglie wavelength.

39.2. IDENTIFY and SET UP: For a photon, $E = \frac{hc}{\lambda}$. For an electron or proton, $p = \frac{h}{\lambda}$ and $E = \frac{p^2}{2m}$, so $E = \frac{h^2}{2m\lambda^2}$.

EXECUTE: (a) $E = \frac{hc}{\lambda} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{0.20\times10^{-9}\text{ m}} = 6.2\text{ keV}$

(b) $E = \frac{h^2}{2m\lambda^2} = \left(\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{0.20\times10^{-9}\text{ m}}\right)^2 \frac{1}{2(9.11\times10^{-31}\text{ kg})} = 6.03\times10^{-18}\text{ J} = 38\text{ eV}$

(c) $E_\text{p} = E_\text{e}\left(\frac{m_\text{e}}{m_\text{p}}\right) = (38\text{ eV})\left(\frac{9.11\times10^{-31}\text{ kg}}{1.67\times10^{-27}\text{ kg}}\right) = 0.021\text{ eV}$

EVALUATE: For a given wavelength a photon has much more energy than an electron, which in turn has more energy than a proton.

39.3. IDENTIFY: For a particle with mass, $\lambda = \frac{h}{p}$ and $K = \frac{p^2}{2m}$.

SET UP: $1\text{ eV} = 1.60\times10^{-19}$ J

EXECUTE: (a) $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(2.80\times10^{-10}\text{ m})} = 2.37\times10^{-24}\text{ kg}\cdot\text{m/s}$.

(b) $K = \frac{p^2}{2m} = \frac{(2.37\times10^{-24}\text{ kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\text{ kg})} = 3.08\times10^{-18}\text{ J} = 19.3\text{ eV}$.

EVALUATE: This wavelength is on the order of the size of an atom. This energy is on the order of the energy of an electron in an atom.

39.4. **IDENTIFY:** For a particle with mass, $\lambda = \dfrac{h}{p}$ and $E = \dfrac{p^2}{2m}$.

SET UP: $1\text{ eV} = 1.60\times10^{-19}\text{ J}$

EXECUTE: $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{\sqrt{2(6.64\times10^{-27}\text{ kg})(4.20\times10^{6}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}} = 7.02\times10^{-15}\text{ m.}$

EVALUATE: This wavelength is on the order of the size of a nucleus.

39.5. **IDENTIFY** and **SET UP:** The de Broglie wavelength is $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$. In the Bohr model, $mvr_n = n(h/2\pi)$, so $mv = nh/(2\pi r_n)$. Combine these two expressions and obtain an equation for λ in terms of n. Then

$\lambda = h\left(\dfrac{2\pi r_n}{nh}\right) = \dfrac{2\pi r_n}{n}$.

EXECUTE: **(a)** For $n = 1, \lambda = 2\pi r_1$ with $r_1 = a_0 = 0.529\times10^{-10}$ m, so

$\lambda = 2\pi(0.529\times10^{-10}\text{ m}) = 3.32\times10^{-10}\text{ m.}$

$\lambda = 2\pi r_1$; the de Broglie wavelength equals the circumference of the orbit.

(b) For $n = 4, \lambda = 2\pi r_4/4$.

$r_n = n^2 a_0$ so $r_4 = 16a_0$.

$\lambda = 2\pi(16a_0)/4 = 4(2\pi a_0) = 4(3.32\times10^{-10}\text{ m}) = 1.33\times10^{-9}\text{ m}$

$\lambda = 2\pi r_4/4$; the de Broglie wavelength is $\dfrac{1}{n} = \dfrac{1}{4}$ times the circumference of the orbit.

EVALUATE: As n increases the momentum of the electron increases and its de Broglie wavelength decreases. For any n, the circumference of the orbits equals an integer number of de Broglie wavelengths.

39.6. **IDENTIFY:** $\lambda = \dfrac{h}{p}$

SET UP: $1\text{ eV} = 1.60\times10^{-19}\text{ J}$. An electron has mass 9.11×10^{-31} kg.

EXECUTE: **(a)** For a nonrelativistic particle, $K = \dfrac{p^2}{2m}$, so $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2Km}}$.

(b) $(6.63\times10^{-34}\text{ J}\cdot\text{s})/\sqrt{2(800\text{ eV})(1.60\times10^{-19}\text{ J/eV})(9.11\times10^{-31}\text{ kg})} = 4.34\times10^{-11}\text{ m.}$

EVALUATE: The de Broglie wavelength decreases when the kinetic energy of the particle increases.

39.7. **IDENTIFY:** A person walking through a door is like a particle going through a slit and hence should exhibit wave properties.

SET UP: The de Broglie wavelength of the person is $\lambda = h/mv$.

EXECUTE: **(a)** Assume $m = 75$ kg and $v = 1.0$ m/s.

$$\lambda = h/mv = (6.626\times10^{-34}\text{ J}\cdot\text{s})/[(75\text{ kg})(1.0\text{ m/s})] = 8.8\times10^{-36}\text{ m}$$

EVALUATE: **(b)** A typical doorway is about 1 m wide, so the person's de Broglie wavelength is much too small to show wave behavior through a "slit" that is about 10^{35} times as wide as the wavelength. Hence ordinary objects do not show wave behavior in everyday life.

39.8. **IDENTIFY** and **SET UP:** Combining Eqs. 37.38 and 37.39 gives $p = mc\sqrt{\gamma^2 - 1}$.

EXECUTE: **(a)** $\lambda = \dfrac{h}{p} = (h/mc)/\sqrt{\gamma^2 - 1} = 4.43\times10^{-12}$ m. (The incorrect nonrelativistic calculation gives 5.05×10^{-12} m.)

(b) $(h/mc)/\sqrt{\gamma^2 - 1} = 7.07\times10^{-13}$ m.

EVALUATE: The de Broglie wavelength decreases when the speed increases.

39.9. **IDENTIFY** and **SET UP:** A photon has zero mass and its energy and wavelength are related by Eq. (38.2). An electron has mass. Its energy is related to its momentum by $E = p^2/2m$ and its wavelength is related to its momentum by Eq. (39.1).

EXECUTE: **(a)** photon: $E = \dfrac{hc}{\lambda}$ so $\lambda = \dfrac{hc}{E} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{(20.0\text{ eV})(1.602\times10^{-19}\text{ J/eV})} = 62.0\text{ nm}.$

electron: $E = p^2/(2m)$ so $p = \sqrt{2mE} =$

$\sqrt{2(9.109\times10^{-31}\text{ kg})(20.0\text{ eV})(1.602\times10^{-19}\text{ J/eV})} = 2.416\times10^{-24}\text{ kg}\cdot\text{m/s}.$ $\lambda = h/p = 0.274\text{ nm}.$

(b) photon: $E = hc/\text{R} = 7.946\times10^{-19}\text{ J} = 4.96\text{ eV}.$

electron: $\lambda = h/p$ so $p = h/\lambda = 2.650\times10^{-27}\text{ kg}\cdot\text{m/s}.$

$E = p^2/(2m) = 3.856\times10^{-24}\text{ J} = 2.41\times10^{-5}\text{ eV}.$

(c) EVALUATE: You should use a probe of wavelength approximately 250 nm. An electron with $\lambda = 250\text{ nm}$ has much less energy than a photon with $\lambda = 250\text{ nm},$ so is less likely to damage the molecule. Note that $\lambda = h/p$ applies to all particles, those with mass and those with zero mass. $E = hf = hc/\lambda$ applies only to photons and $E = p^2/2m$ applies only to particles with mass.

39.10. **IDENTIFY:** Knowing the de Broglie wavelength for an electron, we want to find its speed.

SET UP: $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = 1.00\text{ mm},\ \ m = 9.11\times10^{-31}\text{ kg}.$

EXECUTE: $v = \dfrac{h}{m\lambda} = \dfrac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(9.11\times10^{-31}\text{ kg})(1.00\times10^{-3}\text{ m})} = 0.728\text{ m/s}.$

EVALUATE: Electrons normally move much faster than this, so their de Broglie wavelengths are much much smaller than a millimeter.

39.11. **IDENTIFY** and **SET UP:** Use Eq. (39.1).

EXECUTE: $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(5.00\times10^{-3}\text{ kg})(340\text{ m/s})} = 3.90\times10^{-34}\text{ m}$

EVALUATE: This wavelength is extremely short; the bullet will not exhibit wavelike properties.

39.12. **IDENTIFY:** The kinetic energy of the electron is equal to the energy of the photon. We want to find the wavelengths of each of them.

SET UP: Both for particles with mass (electrons) and for massless particles (photons) the wavelength is related to the momentum p by $\lambda = \dfrac{h}{p}.$ But for each type of particle there is a different expression that relates the energy E and momentum p. For an electron $E = \frac{1}{2}mv^2 = \dfrac{p^2}{2m}$ but for a photon $E = pc.$

EXECUTE: photon: $p = \dfrac{E}{c}$ and $p = \dfrac{h}{\lambda}$ so $\dfrac{h}{\lambda} = \dfrac{E}{c}$ and $\lambda = \dfrac{hc}{E} = \dfrac{1.24\times10^{-6}\text{ eV}\cdot\text{m}}{25\text{ eV}} = 49.6\text{ nm}.$

electron: Solving for p gives $p = \sqrt{2mE}.$ This gives

$p = \sqrt{2(9.11\times10^{-31}\text{ kg})(25\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 2.70\times10^{-24}\text{ kg}\cdot\text{m/s}.$ The wavelength is therefore

$\lambda = \dfrac{h}{p} = \dfrac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{2.70\times10^{-24}\text{ kg}\cdot\text{m/s}} = 0.245\text{ nm}.$

EVALUATE: The wavelengths are quite different. For the electron $\lambda = \dfrac{h}{\sqrt{2mE}}$ and for the photon $\lambda = \dfrac{hc}{E},$ so for an electron λ is proportional to $E^{-1/2}$ and for a photon λ is proportional to E^{-1}. It is *incorrect* to say $p = \dfrac{E}{c}$ for a particle such as an electron that has mass; the correct relation is $p = \dfrac{\sqrt{E^2 - (mc^2)^2}}{c}.$

39.13. **IDENTIFY:** The acceleration gives momentum to the electrons. We can use this momentum to calculate their de Broglie wavelength.

SET UP: The kinetic energy K of the electron is related to the accelerating voltage V by $K = eV$. For an electron $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ and $\lambda = \frac{h}{p}$. For a photon $E = \frac{hc}{\lambda}$.

EXECUTE: **(a)** For an electron $p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{5.00\times10^{-9}\ \text{m}} = 1.33\times10^{-25}\ \text{kg}\cdot\text{m/s}$ and

$E = \frac{p^2}{2m} = \frac{(1.33\times10^{-25}\ \text{kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\ \text{kg})} = 9.71\times10^{-21}\ \text{J}$. $V = \frac{K}{e} = \frac{9.71\times10^{-21}\ \text{J}}{1.60\times10^{-19}\ \text{C}} = 0.0607\ \text{V}$. The electrons would have kinetic energy 0.0607 eV.

(b) $E = \frac{hc}{\lambda} = \frac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{5.00\times10^{-9}\ \text{m}} = 248\ \text{eV}$.

(c) $E = 9.71\times10^{-21}\ \text{J}$

so $\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{9.71\times10^{-21}\ \text{J}} = 20.5\ \mu\text{m}$.

EVALUATE: If they have the same wavelength, the photon has vastly more energy than the electron.

39.14. **IDENTIFY:** $\lambda = \frac{h}{p}$. Apply conservation of energy to relate the potential difference to the speed of the electrons.

SET UP: The mass of an electron is $m = 9.11\times10^{-31}$ kg. The kinetic energy of a photon is $E = \frac{hc}{\lambda}$.

EXECUTE: **(a)** $\lambda = h/mv \rightarrow v = h/m\lambda$. Energy conservation: $e\Delta V = \frac{1}{2}mv^2$.

$$\Delta V = \frac{mv^2}{2e} = \frac{m\left(\frac{h}{m\lambda}\right)^2}{2e} = \frac{h^2}{2em\lambda^2} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.60\times10^{-19}\ \text{C})(9.11\times10^{-31}\ \text{kg})\,(0.15\times10^{-9}\ \text{m})^2} = 66.9\ \text{V}.$$

(b) $E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})\,(3.0\times10^{8}\ \text{m/s})}{0.15\times10^{-9}\ \text{m}} = 1.33\times10^{-15}\ \text{J}$. $e\Delta V = K = E_{\text{photon}}$ and

$\Delta V = \frac{E_{\text{photon}}}{e} = \frac{1.33\times10^{-15}\ \text{J}}{1.6\times10^{-19}\ \text{C}} = 8310\ \text{V}$.

EVALUATE: The electron in part (b) has wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = 0.0135$ nm, much shorter than the wavelength of a photon of the same energy.

39.15. **IDENTIFY:** For an electron, $\lambda = \frac{h}{p}$ and $K = \frac{1}{2}mv^2$. For a photon, $E = \frac{hc}{\lambda}$. The wavelength should be 0.10 nm.

SET UP: For an electron, $m = 9.11\times10^{-31}$ kg.

EXECUTE: **(a)** $\lambda = 0.10$ nm. $p = mv = h/\lambda$ so $v = h/(m\lambda) = 7.3\times10^{6}$ m/s.

(b) $K = \frac{1}{2}mv^2 = 150\ \text{eV}$.

(c) $E = hc/\lambda = 12\ \text{keV}$.

EVALUATE: **(d)** The electron is a better probe because for the same λ it has less energy and is less damaging to the structure being probed.

39.16. **IDENTIFY:** The electrons behave like waves and are diffracted by the slit.

SET UP: We use conservation of energy to find the speed of the electrons, and then use this speed to find their de Broglie wavelength, which is $\lambda = h/mv$. Finally we know that the first dark fringe for single-slit diffraction occurs when $a \sin\theta = \lambda$.

EXECUTE: **(a)** Use energy conservation to find the speed of the electron: $\frac{1}{2}mv^2 = eV$.

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\ \text{C})(100\ \text{V})}{9.11\times10^{-31}\ \text{kg}}} = 5.93\times10^{6}\ \text{m/s}$$

which is about 2% the speed of light, so we can ignore relativity.

(b) First find the de Broglie wavelength:

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(5.93\times10^{6}\ \text{m/s})} = 1.23\times10^{-10}\ \text{m} = 0.123\ \text{nm}$$

For the first single-slit dark fringe, we have $a \sin\theta = \lambda$, which gives

$$a = \frac{\lambda}{\sin\theta} = \frac{1.23\times10^{-10}\ \text{m}}{\sin(11.5°)} = 6.16\times10^{-10}\ \text{m} = 0.616\ \text{nm}$$

EVALUATE: The slit width is around 5 times the de Broglie wavelength of the electron, and both are much smaller than the wavelength of visible light.

39.17. **IDENTIFY:** The intensity maxima are located by Eq. (39.4). Use $\lambda = \frac{h}{p}$ for the wavelength of the neutrons. For a particle, $p = \sqrt{2mE}$.

SET UP: For a neutron, $m = 1.67\times10^{-27}$ kg.

EXECUTE: For $m = 1$, $\lambda = d\sin\theta = \frac{h}{\sqrt{2mE}}$.

$$E = \frac{h^2}{2md^2\sin^2\theta} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.675\times10^{-27}\ \text{kg})\,(9.10\times10^{-11}\ \text{m})^2\sin^2(28.6°)} = 6.91\times10^{-20}\ \text{J} = 0.432\ \text{eV}.$$

EVALUATE: The neutrons have $\lambda = 0.0436$ nm, comparable to the atomic spacing.

39.18. **IDENTIFY:** Intensity maxima occur when $d\sin\theta = m\lambda$. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2ME}}$ so $d\sin\theta = \frac{mh}{\sqrt{2ME}}$.

SET UP: Here m is the order of the maxima, whereas M is the mass of the incoming particle.

EXECUTE: **(a)** $d = \frac{mh}{\sqrt{2ME}\sin\theta} = \frac{(2)(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(188\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}\sin(60.6°)} =$

$2.06\times10^{-10}\ \text{m} = 0.206\ \text{nm}$.

(b) $m = 1$ also gives a maximum.

$\theta = \arcsin\left(\frac{(1)(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(188\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}(2.06\times10^{-10}\ \text{m})}\right) = 25.8°$. This is the only other one. If we let $m \geq 3$, then there are no more maxima.

(c) $E = \frac{m^2h^2}{2Md^2\sin^2\theta} = \frac{(1)^2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})\,(2.60\times10^{-10}\ \text{m})^2\sin^2(60.6°)}$.

$= 7.49\times10^{-18}\ \text{J} = 46.8\ \text{eV}$.

Using this energy, if we let $m = 2$, then $\sin\theta > 1$. Thus, there is no $m = 2$ maximum in this case.

EVALUATE: As the energy of the electrons is lowered their wavelength increases and a given intensity maximum occurs at a larger angle.

39.19. **IDENTIFY:** The condition for a maximum is $d\sin\theta = m\lambda$. $\lambda = \dfrac{h}{p} = \dfrac{h}{Mv}$, so $\theta = \arcsin\left(\dfrac{mh}{dMv}\right)$.

SET UP: Here *m* is the order of the maximum, whereas *M* is the incoming particle mass.

EXECUTE: **(a)** $m = 1 \Rightarrow \theta_1 = \arcsin\left(\dfrac{h}{dMv}\right)$

$$= \arcsin\left(\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(1.60\times10^{-6}\ \text{m})(9.11\times10^{-31}\ \text{kg})(1.26\times10^{4}\ \text{m/s})}\right) = 2.07°.$$

$$m = 2 \Rightarrow \theta_2 = \arcsin\left(\frac{(2)(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1.60\times10^{-6}\ \text{m})(9.11\times10^{-31}\ \text{kg})(1.26\times10^{4}\ \text{m/s})}\right) = 4.14°.$$

(b) For small angles (in radians!) $y \cong D\theta$, so $y_1 \approx (50.0\ \text{cm})\ (2.07°)\left(\dfrac{\pi\ \text{radians}}{180°}\right) = 1.81\ \text{cm}$,

$y_2 \approx (50.0\ \text{cm})\ (4.14°)\left(\dfrac{\pi\ \text{radians}}{180°}\right) = 3.61\ \text{cm}$ and $y_2 - y_1 = 3.61\ \text{cm} - 1.81\ \text{cm} = 1.80\ \text{cm}$.

EVALUATE: For these electrons, $\lambda = \dfrac{h}{mv} = 0.0577\ \mu\text{m}$. λ is much less than *d* and the intensity maxima occur at small angles.

39.20. **IDENTIFY:** $\lambda = \dfrac{h}{p}$. Conservation of energy gives $eV = K = \dfrac{p^2}{2m}$, where *V* is the accelerating voltage.

SET UP: The electron mass is 9.11×10^{-31} kg and the proton mass is 1.67×10^{-27} kg.

EXECUTE: **(a)** $eV = K = \dfrac{p^2}{2m} = \dfrac{(h/\lambda)^2}{2m}$, so $V = \dfrac{(h/\lambda)^2}{2me} = 419\ \text{V}$.

(b) The voltage is reduced by the ratio of the particle masses, $(419\ \text{V})\dfrac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}} = 0.229\ \text{V}$.

EVALUATE: $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$. For the same λ, particles of greater mass have smaller *E*, so a smaller accelerating voltage is needed for protons.

39.21. **IDENTIFY** and **SET UP:** For a photon $E_{\text{ph}} = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$. For an electron

$$E_{\text{e}} = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2 = \frac{h^2}{2m\lambda^2}.$$

EXECUTE: **(a)** photon $E_{\text{ph}} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{10.0\times10^{-9}\ \text{m}} = 1.99\times10^{-17}\ \text{J}$

electron $E_{\text{e}} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})(10.0\times10^{-9}\ \text{m})^2} = 2.41\times10^{-21}\ \text{J}$

$$\frac{E_{\text{ph}}}{E_{\text{e}}} = \frac{1.99\times10^{-17}\ \text{J}}{2.41\times10^{-21}\ \text{J}} = 8.26\times10^{3}$$

(b) The electron has much less energy so would be less damaging.

EVALUATE: For a particle with mass, such as an electron, $E \sim \lambda^{-2}$. For a massless photon $E \sim \lambda^{-1}$.

39.22. **IDENTIFY:** The kinetic energy of the alpha particle is all converted to electrical potential energy at closest approach. The force on the alpha particle is the electrical repulsion of the nucleus.

SET UP: The electrical potential energy of the system is $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$ and the kinetic energy is $K = \frac{1}{2}mv^2$. The electrical force is $R = 2.5$ m (at closest approach).

(a) Equating the initial kinetic energy and the final potential energy and solving for the separation radius r gives

$$r = \frac{1}{4\pi\varepsilon_0}\frac{(92e)(2e)}{K} = \frac{1}{4\pi\varepsilon_0}\frac{(184)(1.60\times10^{-19}\text{ C})^2}{(4.78\times10^6\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 5.54\times10^{-14}\text{ m}.$$

(b) The above result may be substituted into Coulomb's law. Alternatively, the relation between the magnitude of the force and the magnitude of the potential energy in a Coulomb field is $F = \frac{|U|}{r}$. $|U| = K$, so $F = \frac{K}{r} = \frac{(4.78\times10^6\text{ eV})(1.6\times10^{-19}\text{ J/ev})}{(5.54\times10^{-14}\text{ m})} = 13.8\text{ N}.$

EVALUATE: The result in part (a) is comparable to the radius of a large nucleus, so it is reasonable. The force in part (b) is around 3 pounds, which is large enough to be easily felt by a person.

39.23. **(a) IDENTIFY:** If the particles are treated as point charges, $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$.

SET UP: $q_1 = 2e$ (alpha particle); $q_2 = 82e$ (gold nucleus); r is given so we can solve for U.

EXECUTE: $U = (8.987\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(2)(82)(1.602\times10^{-19}\text{ C})^2}{6.50\times10^{-14}\text{ m}} = 5.82\times10^{-13}\text{ J}$

$U = 5.82\times10^{-13}\text{ J }(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 3.63\times10^6\text{ eV} = 3.63\text{ MeV}$

(b) IDENTIFY: Apply conservation of energy: $K_1 + U_1 = K_2 + U_2$.

SET UP: Let point 1 be the initial position of the alpha particle and point 2 be where the alpha particle momentarily comes to rest. Alpha particle is initially far from the lead nucleus implies $r_1 \approx \infty$ and $U_1 = 0$. Alpha particle stops implies $K_2 = 0$.

EXECUTE: Conservation of energy thus says $K_1 = U_2 = 5.82\times10^{-13}\text{ J} = 3.63\text{ MeV}$.

(c) $K = \frac{1}{2}mv^2$ so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.82\times10^{-13}\text{ J})}{6.64\times10^{-27}\text{ kg}}} = 1.32\times10^7\text{ m/s}$

EVALUATE: $v/c = 0.044$, so it is ok to use the nonrelativistic expression to relate K and v. When the alpha particle stops, all its initial kinetic energy has been converted to electrostatic potential energy.

39.24. **IDENTIFY:** The minimum energy the photon would need is the 3.84 eV bond strength.

SET UP: The photon energy $E = hf = \frac{hc}{\lambda}$ must equal the bond strength.

EXECUTE: $\frac{hc}{\lambda} = 3.80\text{ eV}$, so $\lambda = \frac{hc}{3.80\text{ eV}} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^8\text{ m/s})}{3.80\text{ eV}} = 327\text{ nm}.$

EVALUATE: Any photon having a shorter wavelength would also spell doom for the Horta!

39.25. **IDENTIFY** and **SET UP:** Use the energy to calculate n for this state. Then use the Bohr equation, Eq. (39.6), to calculate L.

EXECUTE: $E_n = -(13.6\text{ eV})/n^2$, so this state has $n = \sqrt{13.6/1.51} = 3$. In the Bohr model, $L = n\hbar$ so for this state $L = 3\hbar = 3.16\times10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}$.

EVALUATE: We will find in Section 41.1 that the modern quantum mechanical description gives a different result.

39.26. **IDENTIFY** and **SET UP:** For a hydrogen atom $E_n = -\frac{13.6 \text{ eV}}{n^2}$. $\Delta E = \frac{hc}{\lambda}$, where ΔE is the magnitude of the energy change for the atom and λ is the wavelength of the photon that is absorbed or emitted.

EXECUTE: $\Delta E = E_4 - E_1 = -(13.6 \text{ eV})\left(\frac{1}{4^2} - \frac{1}{1^2}\right) = +12.75 \text{ eV}.$

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15} \text{ eV}\cdot\text{s})(3.00\times10^8 \text{ m/s})}{12.75 \text{ eV}} = 97.3 \text{ nm}. \ f = \frac{c}{\lambda} = 3.08\times10^{15} \text{ Hz}.$$

EVALUATE: This photon is in the ultraviolet region of the electromagnetic spectrum.

39.27. **IDENTIFY:** The force between the electron and the nucleus in Be^{3+} is $F = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}$, where $Z = 4$ is the nuclear charge. All the equations for the hydrogen atom apply to Be^{3+} if we replace e^2 by Ze^2.

(a) SET UP: Modify Eq. (39.14).

EXECUTE: $E_n = -\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2}$ (hydrogen) becomes

$$E_n = -\frac{1}{\epsilon_0^2}\frac{m(Ze^2)^2}{8n^2h^2} = Z^2\left(-\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2}\right) = Z^2\left(-\frac{13.60 \text{ eV}}{n^2}\right) (\text{for Be}^{3+})$$

The ground-level energy of Be^{3+} is $E_1 = 16\left(-\frac{13.60 \text{ eV}}{1^2}\right) = -218 \text{ eV}.$

EVALUATE: The ground-level energy of Be^{3+} is $Z^2 = 16$ times the ground-level energy of H.

(b) SET UP: The ionization energy is the energy difference between the $n \to \infty$ level energy and the $n = 1$ level energy.

EXECUTE: The $n \to \infty$ level energy is zero, so the ionization energy of Be^{3+} is 218 eV.

EVALUATE: This is 16 times the ionization energy of hydrogen.

(c) SET UP: $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ just as for hydrogen but now R has a different value.

EXECUTE: $R_H = \frac{me^4}{8\epsilon_0^2h^3c} = 1.097\times10^7 \text{ m}^{-1}$ for hydrogen becomes

$$R_{Be} = Z^2\frac{me^4}{8\epsilon_0^2h^3c} = 16(1.097\times10^7 \text{ m}^{-1}) = 1.755\times10^8 \text{ m}^{-1} \text{ for Be}^{3+}.$$

For $n = 2$ to $n = 1, \frac{1}{\lambda} = R_{Be}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 3R_{Be}/4.$

$$\lambda = 4/(3R_{Be}) = 4/(3(1.755\times10^8 \text{ m}^{-1})) = 7.60\times10^{-9} \text{ m} = 7.60 \text{ nm}.$$

EVALUATE: This wavelength is smaller by a factor of 16 compared to the wavelength for the corresponding transition in the hydrogen atom.

(d) SET UP: Modify Eq. (39.8): $r_n = \epsilon_0\frac{n^2h^2}{\pi me^2}$ (hydrogen).

EXECUTE: $r_n = \epsilon_0\frac{n^2h^2}{\pi m(Ze^2)}$ (Be^{3+}).

EVALUATE: For a given n the orbit radius for Be^{3+} is smaller by a factor of $Z = 4$ compared to the corresponding radius for hydrogen.

39.28. **IDENTIFY** and **SET UP:** $E_n = -\dfrac{13.6\text{ eV}}{n^2}.$

EXECUTE: **(a)** $E_n = -\dfrac{13.6\text{ eV}}{n^2}$ and $E_{n+1} = -\dfrac{13.6\text{ eV}}{(n+1)^2}.$

$$\Delta E = E_{n+1} - E_n = (-13.6\text{ eV})\left[\frac{1}{(n+1)^2} - \frac{1}{n^2}\right] = -(13.6\text{ eV})\frac{n^2-(n+1)^2}{(n^2)(n+1)^2}.\ \Delta E = (13.6\text{ eV})\frac{2n+1}{(n^2)(n+1)^2}.$$

As n becomes large, $\Delta E \to (13.6\text{ eV})\dfrac{2n}{n^4} = (13.6\text{ eV})\dfrac{2}{n^3}$. Thus ΔE becomes small as n becomes large.

(b) $r_n = n^2 r_1$ so the orbits get farther apart in space as n increases.

EVALUATE: There are an infinite number of bound levels for the hydrogen atom. As n increases the energies of the bound levels converge to the ionization threshold.

39.29. **IDENTIFY:** Apply Eqs. (39.8) and (39.9).

SET UP: The orbital period for state n is the circumference of the orbit divided by the orbital speed.

EXECUTE: **(a)** $v_n = \dfrac{1}{\epsilon_0}\dfrac{e^2}{2nh} : n = 1 \Rightarrow v_1 = \dfrac{(1.60\times10^{-19}\text{ C})^2}{\epsilon_0 2\,(6.63\times10^{-34}\text{ J}\cdot\text{s})} = 2.18\times10^6\text{ m/s}.$

$n = 2 \Rightarrow v_2 = \dfrac{v_1}{2} = 1.09\times10^6\text{ m/s}.\ n = 3 \Rightarrow v_3 = \dfrac{v_1}{3} = 7.27\times10^5\text{ m/s}.$

(b) Orbital period $= \dfrac{2\pi r_n}{v_n} = \dfrac{2\epsilon_0 n^2 h^2/me^2}{1/\epsilon_0 \cdot e^2/2nh} = \dfrac{4\epsilon_0^2 n^3 h^3}{me^4}.$

$$n = 1 \Rightarrow T_1 = \frac{4\epsilon_0^2(6.63\times10^{-34}\text{ J}\cdot\text{s})^3}{(9.11\times10^{-31}\text{ kg})(1.60\times10^{-19}\text{ C})^4} = 1.53\times10^{-16}\text{ s}$$

$n = 2\text{: } T_2 = T_1(2)^3 = 1.22\times10^{-15}\text{ s}.\ n = 3\text{: } T_3 = T_1(3)^3 = 4.13\times10^{-15}\text{ s}.$

(c) number of orbits $= \dfrac{1.0\times10^{-8}\text{s}}{1.22\times10^{-15}\text{ s}} = 8.2\times10^6.$

EVALUATE: The orbital speed is proportional to $1/n$, the orbital radius is proportional to n^2, and the orbital period is proportional to n^3.

39.30. **IDENTIFY** and **SET UP:** The ionization threshold is at $E = 0$. The energy of an absorbed photon equals the energy gained by the atom and the energy of an emitted photon equals the energy lost by the atom.

EXECUTE: **(a)** $\Delta E = 0 - (-20\text{ eV}) = 20\text{ eV}$

(b) When the atom in the $n = 1$ level absorbs an 18-eV photon, the final level of the atom is $n = 4$. The possible transitions from $n = 4$ and corresponding photon energies are $n = 4 \to n = 3$, 3 eV; $n = 4 \to n = 2$, 8 eV; $n = 4 \to n = 1$, 18 eV. Once the atom has gone to the $n = 3$ level, the following transitions can occur: $n = 3 \to n = 2$, 5 eV; $n = 3 \to n = 1$, 15 eV. Once the atom has gone to the $n = 2$ level, the following transition can occur: $n = 2 \to n = 1$, 10 eV. The possible energies of emitted photons are: 3 eV, 5 eV, 8 eV, 10 eV, 15 eV and 18 eV.

(c) There is no energy level 8 eV higher in energy than the ground state, so the photon cannot be absorbed.

(d) The photon energies for $n = 3 \to n = 2$ and for $n = 3 \to n = 1$ are 5 eV and 15 eV. The photon energy for $n = 4 \to n = 3$ is 3 eV. The work function must have a value between 3 eV and 5 eV.

EVALUATE: The atom has discrete energy levels, so the energies of emitted or absorbed photons have only certain discrete energies.

39.31. **IDENTIFY** and **SET UP:** The wavelength of the photon is related to the transition energy $E_i - E_f$ of the atom by $E_i - E_f = \frac{hc}{\lambda}$ where $hc = 1.240 \times 10^{-6}$ eV · m.

EXECUTE: **(a)** The minimum energy to ionize an atom is when the upper state in the transition has $E = 0$, so $E_1 = -17.50$ eV. For $n = 5 \to n = 1$, $\lambda = 73.86$ nm and $E_5 - E_1 = \frac{1.240 \times 10^{-6}\ \text{eV} \cdot \text{m}}{73.86 \times 10^{-9}\ \text{m}} = 16.79$ eV.

$E_5 = -17.50\ \text{eV} + 16.79\ \text{eV} = -0.71$ eV. For $n = 4 \to n = 1$, $\lambda = 75.63$ nm and $E_4 = -1.10$ eV. For $n = 3 \to n = 1$, $\lambda = 79.76$ nm and $E_3 = -1.95$ eV. For $n = 2 \to n = 1$, $\lambda = 94.54$ nm and $E_2 = -4.38$ eV.

(b) $E_i - E_f = E_4 - E_2 = -1.10\ \text{eV} - (-4.38\ \text{eV}) = 3.28$ eV and $\lambda = \frac{hc}{E_i - E_f} = \frac{1.240 \times 10^{-6}\ \text{eV} \cdot \text{m}}{3.28\ \text{eV}} = 378$ nm

EVALUATE: The $n = 4 \to n = 2$ transition energy is smaller than the $n = 4 \to n = 1$ transition energy so the wavelength is longer. In fact, this wavelength is longer than for any transition that ends in the $n = 1$ state.

39.32. **IDENTIFY** and **SET UP:** For the Lyman series the final state is $n = 1$ and the wavelengths are given by $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, $n = 2, 3, \ldots$. For the Paschen series the final state is $n = 3$ and the wavelengths are given by $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, $n = 4, 5, \ldots$. $R = 1.097 \times 10^7\ \text{m}^{-1}$. The longest wavelength is for the smallest *n* and the shortest wavelength is for $n \to \infty$.

EXECUTE: Lyman: Longest: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$. $\lambda = \frac{4}{3(1.097 \times 10^7\ \text{m}^{-1})} = 121.5$ nm.

Shortest: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R$. $\lambda = \frac{1}{1.097 \times 10^7\ \text{m}^{-1}} = 91.16$ nm

Paschen: Longest: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7R}{144}$. $\lambda = \frac{144}{7(1.097 \times 10^7\ \text{m}^{-1})} = 1875$ nm.

Shortest: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = \frac{R}{9}$.

EVALUATE: The Lyman series is in the ultraviolet. The Paschen series is in the infrared.

39.33. **IDENTIFY:** Apply conservation of energy to the system of atom and photon.

SET UP: The energy of a photon is $E_\gamma = \frac{hc}{\lambda}$.

EXECUTE: **(a)** $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})(3.00 \times 10^8\ \text{m/s})}{8.60 \times 10^{-7}\ \text{m}} = 2.31 \times 10^{-19}\ \text{J} = 1.44$ eV. So the internal energy of the atom increases by 1.44 eV to $E = -6.52\ \text{eV} + 1.44\ \text{eV} = -5.08$ eV.

(b) $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})(3.00 \times 10^8\ \text{m/s})}{4.20 \times 10^{-7}\ \text{m}} = 4.74 \times 10^{-19}\ \text{J} = 2.96$ eV. So the final internal energy of the atom decreases to $E = -2.68\ \text{eV} - 2.96\ \text{eV} = -5.64$ eV.

EVALUATE: When an atom absorbs a photon the energy of the atom increases. When an atom emits a photon the energy of the atom decreases.

39.34. **IDENTIFY** and **SET UP:** Balmer's formula is $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$. For the H_γ spectral line $n = 5$. Once we have λ, calculate *f* from $f = c/\lambda$ and *E* from Eq. (38.2).

EXECUTE: **(a)** $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = R\left(\frac{25 - 4}{100}\right) = R\left(\frac{21}{100}\right)$.

Thus $\lambda = \dfrac{100}{21R} = \dfrac{100}{21(1.097\times10^{7})}\ \text{m} = 4.341\times10^{-7}\ \text{m} = 434.1\ \text{nm}.$

(b) $f = \dfrac{c}{\lambda} = \dfrac{2.998\times10^{8}\ \text{m/s}}{4.341\times10^{-7}\ \text{m}} = 6.906\times10^{14}\ \text{Hz}$

(c) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(6.906\times10^{14}\ \text{Hz}) = 4.576\times10^{-19}\ \text{J} = 2.856\ \text{eV}$

EVALUATE: Section 39.3 shows that the longest wavelength in the Balmer series (H_α) is 656 nm and the shortest is 365 nm. Our result for H_γ falls within this range. The photon energies for hydrogen atom transitions are in the eV range, and our result is of this order.

39.35. **IDENTIFY:** We know the power of the laser beam, so we know the energy per second that it delivers. The wavelength of the light tells us the energy of each photon, so we can use that to calculate the number of photons delivered per second.

SET UP: The energy of each photon is $E = hf = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon.

EXECUTE: $\lambda = 10.6\times10^{-6}$ m, so $E = 1.88\times10^{-20}$ J. $P = \dfrac{E_{\text{tot}}}{t} = \dfrac{NE}{t}$ so

$\dfrac{N}{t} = \dfrac{P}{E} = \dfrac{0.100\times10^{3}\ \text{W}}{1.88\times10^{-20}\ \text{J}} = 5.32\times10^{21}$ photons/s.

EVALUATE: At over 10^{21} photons per second, we can see why we do not detect individual photons.

39.36. **IDENTIFY:** We can calculate the energy of a photon from its wavelength. Knowing the intensity of the beam and the energy of a single photon, we can determine how many photons strike the blemish with each pulse.

SET UP: The energy of each photon is $E = hf = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon. The photon beam is spread over an area $A = \pi r^2$ with $r = 2.5$ mm.

EXECUTE: **(a)** $\lambda = 585$ nm and $E = \dfrac{hc}{\lambda} = 3.40\times10^{-19}\ \text{J} = 2.12\ \text{eV}.$

(b) $P = \dfrac{E_{\text{tot}}}{t} = \dfrac{NE}{t}$ so $N = \dfrac{Pt}{E} = \dfrac{(20.0\ \text{W})(0.45\times10^{-3}\ \text{s})}{3.40\times10^{-19}\ \text{J}} = 2.65\times10^{16}$ photons. These photons are spread over an area πr^2, so the number of photons per mm^2 is $\dfrac{2.65\times10^{16}\ \text{photons}}{\pi(2.5\ \text{mm})^2} = 1.35\times10^{15}\ \text{photons/mm}^2.$

EVALUATE: With so many photons per mm^2, it is impossible to detect individual photons.

39.37. **IDENTIFY** and **SET UP:** The number of photons emitted each second is the total energy emitted divided by the energy of one photon. The energy of one photon is given by Eq. (38.2). $E = Pt$ gives the energy emitted by the laser in time t.

EXECUTE: In 1.00 s the energy emitted by the laser is $(7.50\times10^{-3}\ \text{W})(1.00\ \text{s}) = 7.50\times10^{-3}\ \text{J}.$

The energy of each photon is $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{10.6\times10^{-6}\ \text{m}} = 1.874\times10^{-20}\ \text{J}.$

Therefore $\dfrac{7.50\times10^{-3}\ \text{J/s}}{1.874\times10^{-20}\ \text{J/photon}} = 4.00\times10^{17}$ photons/s

EVALUATE: The number of photons emitted per second is extremely large.

39.38. **IDENTIFY** and **SET UP:** Visible light has wavelengths from about 400 nm to about 700 nm. The energy of each photon is $E = hf = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon.

EXECUTE: **(a)** 193 nm is shorter than visible light so is in the ultraviolet.

(b) $E = \dfrac{hc}{\lambda} = 1.03\times10^{18}\ \text{J} = 6.44\ \text{eV}$

(c) $P = \dfrac{E_{\text{tot}}}{t} = \dfrac{NE}{t}$ so $N = \dfrac{Pt}{E} = \dfrac{(1.50\times10^{-3}\ \text{W})(12.0\times10^{-9}\ \text{s})}{1.03\times10^{-18}\ \text{J}} = 1.75\times10^{7}$ photons

EVALUATE: A very small amount of energy is delivered to the lens in each pulse, but this still corresponds to a large number of photons.

39.39. **IDENTIFY:** Apply Eq. (39.18): $\dfrac{n_{5s}}{n_{3p}} = e^{-(E_{5s}-E_{3p})/kT}$

SET UP: $E_{5s} = 20.66\ \text{eV}$ and $E_{3p} = 18.70\ \text{eV}$

EXECUTE: $E_{5s} - E_{3p} = 20.66\ \text{eV} - 18.70\ \text{eV} = 1.96\ \text{eV}(1.602\times10^{-19}\ \text{J/1 eV}) = 3.140\times10^{-19}\ \text{J}$

(a) $\dfrac{n_{5s}}{n_{3p}} = e^{-(3.140\times10^{-19}\ \text{J})/[(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})]} = e^{-75.79} = 1.2\times10^{-33}$

(b) $\dfrac{n_{5s}}{n_{3p}} = e^{-(3.140\times10^{-19}\ \text{J})/[(1.38\times10^{-23}\ \text{J/K})(600\ \text{K})]} = e^{-37.90} = 3.5\times10^{-17}$

(c) $\dfrac{n_{5s}}{n_{3p}} = e^{-(3.140\times10^{-19}\ \text{J})/[(1.38\times10^{-23}\ \text{J/K})(1200\ \text{K})]} = e^{-18.95} = 5.9\times10^{-9}$

(d) EVALUATE: At each of these temperatures the number of atoms in the 5*s* excited state, the initial state for the transition that emits 632.8 nm radiation, is quite small. The ratio increases as the temperature increases.

39.40. **IDENTIFY:** Apply Eq. (39.18).

SET UP: The energy of each of these excited states above the ground state is hc/λ, where λ is the wavelength of the photon emitted in the transition from the excited state to the ground state.

EXECUTE: $\dfrac{n_{2P_{3/2}}}{n_{2P_{1/2}}} = e^{-(E_{2P3/2} - E_{2P1/2})/KT}$. From the diagram

$$\Delta E_{3/2-\text{g}} = \frac{hc}{\lambda_1} = \frac{(6.626\times10^{-34}\ \text{J})(2.998\times10^{8}\ \text{m/s})}{5.890\times10^{-7}\ \text{m}} = 3.373\times10^{-19}\ \text{J}.$$

$\Delta E_{1/2-\text{g}} = \dfrac{hc}{\lambda_2} = \dfrac{(6.626\times10^{-34}\ \text{J})(2.998\times10^{8}\ \text{m/s})}{5.896\times10^{-7}\ \text{m}} = 3.369\times10^{-19}\ \text{J}$. So $\Delta E_{3/2-1/2} =$ $3.373\times10^{-19}\ \text{J} - 3.369\times10^{-19}\ \text{J} = 4.00\times10^{-22}\ \text{J}.$

$\dfrac{n_{2P_{3/2}}}{n_{2P_{1/2}}} = e^{-(4.00\times10^{-22}\ \text{J})/(1.38\times10^{-23}\text{J/K}\cdot 500\ \text{K})} = 0.944.$ So more atoms are in the $2P_{1/2}$ state.

EVALUATE: At this temperature $kT = 6.9\times10^{-21}\ \text{J}$. This is greater than the energy separation between the states, so an atom has almost equal probability for being in either state, with only a small preference for the lower energy state.

39.41. **IDENTIFY:** Energy radiates at the rate $H = Ae\sigma T^4$.

SET UP: The surface area of a cylinder of radius r and length l is $A = 2\pi rl$.

EXECUTE: **(a)** $T = \left(\dfrac{H}{Ae\sigma}\right)^{1/4} = \left(\dfrac{100\ \text{W}}{2\pi(0.20\times10^{-3}\ \text{m})(0.30\ \text{m})(0.26)(5.671\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}\right)^{1/4}.$

$T = 2.06\times10^{3}\ \text{K}.$

(b) $\lambda_m T = 2.90 \times 10^{-3}\ \text{m} \cdot \text{K};\ \lambda_m = 1410\ \text{nm}.$

EVALUATE: (c) λ_m is in the infrared. The incandescent bulb is not a very efficient source of visible light because much of the emitted radiation is in the infrared.

39.42. IDENTIFY: Apply Eq. (39.21) and $c = f\lambda$.

SET UP: T in kelvins gives λ in meters.

EXECUTE: (a) $\lambda_m = \dfrac{2.90 \times 10^{-3}\ \text{m} \cdot \text{K}}{3.00\ \text{K}} = 0.966\ \text{mm}$, and $f = \dfrac{c}{\lambda_m} = 3.10 \times 10^{11}\ \text{Hz}.$

(b) A factor of 100 increase in the temperature lowers λ_m by a factor of 100 to 9.66 μm and raises the frequency by the same factor, to 3.10×10^{13} Hz.

(c) Similarly, $\lambda_m = 966\ \text{nm}$ and $f = 3.10 \times 10^{14}\ \text{Hz}.$

EVALUATE: λ_m decreases when T increases, as explained in the textbook.

39.43. IDENTIFY and **SET UP:** The wavelength λ_m where the Planck distribution peaks is given by Eq. (39.21).

EXECUTE: $\lambda_m = \dfrac{2.90 \times 10^{-3}\ \text{m} \cdot \text{K}}{2.728\ \text{K}} = 1.06 \times 10^{-3}\ \text{m} = 1.06\ \text{mm}.$

EVALUATE: This wavelength is in the microwave portion of the electromagnetic spectrum. This radiation is often referred to as the "microwave background" (Section 44.7). Note that in Eq. (39.21), T must be in kelvins.

39.44. IDENTIFY and **SET UP:** Apply Eq. (39.21).

EXECUTE: $T = \dfrac{2.90 \times 10^{-3}\ \text{m} \cdot \text{K}}{\lambda_m} = \dfrac{2.90 \times 10^{-3}\ \text{m} \cdot \text{K}}{400 \times 10^{-9}\ \text{m}} = 7.25 \times 10^{3}\ \text{K}.$

EVALUATE: 400 nm = 0.4 μm. This is shorter than any of the λ_m values shown in Figure 39.32 in the textbook, and the temperature is therefore higher than those in the figure.

39.45. IDENTIFY: Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law and Wien's displacement law.

SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$. Wien's displacement law tells us that the peak-intensity wavelength is $\lambda_m = (\text{constant})/T$.

EXECUTE: (a) The hot and cool stars radiate the same total power, so the Stefan-Boltzmann law gives $\sigma A_h T_h^4 = \sigma A_c T_c^4 \Rightarrow 4\pi R_h^2 T_h^4 = 4\pi R_c^2 T_c^4 = 4\pi (3R_h)^2 T_c^4 \Rightarrow T_h^4 = 9T^4 \Rightarrow T_h = T\sqrt{3} = 1.7T$, rounded to two significant digits.

(b) Using Wien's law, we take the ratio of the wavelengths, giving

$$\frac{\lambda_m(\text{hot})}{\lambda_m(\text{cool})} = \frac{T_c}{T_h} = \frac{T}{T\sqrt{3}} = \frac{1}{\sqrt{3}} = 0.58,$$ rounded to two significant digits.

EVALUATE: Although the hot star has only 1/9 the surface area of the cool star, its absolute temperature has to be only 1.7 times as great to radiate the same amount of energy.

39.46. IDENTIFY: Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law.

SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: (a) $I = \sigma T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(24{,}000\ \text{K})^4 = 1.9 \times 10^{10}\ \text{W/m}^2$

(b) Wien's law gives $\lambda_m = (0.00290\ \text{m} \cdot \text{K})/(24{,}000\ \text{K}) = 1.2 \times 10^{-7}\ \text{m} = 20\ \text{nm}$

This is not visible since the wavelength is less than 400 nm.

(c) $P = AI \Rightarrow 4\pi R^2 = P/I = (1.00 \times 10^{25}\ \text{W})/(1.9 \times 10^{10}\ \text{W/m}^2)$

which gives $R_{\text{Sirius}} = 6.51 \times 10^{6}\ \text{m} = 6510\ \text{km}.$

$R_{\text{Sirius}}/R_{\text{sun}} = (6.51\times10^6\text{ m})/(6.96\times10^9\text{ m}) = 0.0093$, which gives

$$R_{\text{Sirius}} = 0.0093\,R_{\text{sun}} \approx 1\%\,R_{\text{sun}}$$

(d) Using the Stefan-Boltzmann law, we have

$$\frac{P_{\text{sun}}}{P_{\text{Sirius}}} = \frac{\sigma A_{\text{sun}}T_{\text{sun}}^4}{\sigma A_{\text{Sirius}}T_{\text{Sirius}}^4} = \frac{4\pi R_{\text{sun}}^2 T_{\text{sun}}^4}{4\pi R_{\text{Sirius}}^2 T_{\text{Sirius}}^4} = \left(\frac{R_{\text{sun}}}{R_{\text{Sirius}}}\right)^2\left(\frac{T_{\text{sun}}}{T_{\text{Sirius}}}\right)^4 \cdot \frac{P_{\text{sun}}}{P_{\text{Sirius}}} = \left(\frac{R_{\text{sun}}}{0.00935R_{\text{sun}}}\right)^2\left(\frac{5800\text{ K}}{24{,}000\text{ K}}\right)^4 = 39$$

EVALUATE: Even though the absolute surface temperature of Sirius B is about 4 times that of our sun, it radiates only 1/39 times as much energy per second as our sun because it is so small.

39.47. **IDENTIFY:** Apply the Wien displacement law to relate λ_{m} and T. Apply the Stefan-Boltzmann law to relate the power output of the star to its surface area and therefore to its radius.

SET UP: For a sphere $A = 4\pi r^2$. Since we assume a blackbody, $e = 1$.

EXECUTE: **(a)** Wien's law: $\lambda_{\text{m}} = \dfrac{k}{T}$. $\lambda_{\text{m}} = \dfrac{2.90\times10^{-3}\text{ K}\cdot\text{m}}{30{,}000\text{ K}} = 9.7\times10^{-8}\text{ m} = 97\text{ nm}$. This peak is in the ultraviolet region, which is *not* visible. The star is blue because the largest part of the visible light radiated is in the blue/violet part of the visible spectrum.

(b) $P = \sigma A T^4$ (Stefan-Boltzmann law)

$$(100{,}000)(3.86\times10^{26}\text{ W}) = \left(5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\text{K}^4}\right)(4\pi R^2)(30{,}000\text{ K})^4$$

$$R = 8.2\times10^9\text{ m}$$

$$R_{\text{star}}/R_{\text{sun}} = \frac{8.2\times10^9\text{ m}}{6.96\times10^8\text{ m}} = 12$$

EVALUATE: **(c)** The visual luminosity is proportional to the power radiated at visible wavelengths. Much of the power is radiated nonvisible wavelengths, which does not contribute to the visible luminosity.

39.48. **IDENTIFY:** Since we know only that the mosquito is somewhere in the room, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is an uncertainty in its momentum.

SET UP: The uncertainty principle is $\Delta x\Delta p_x \ge \hbar/2$.

EXECUTE: **(a)** You know the mosquito is somewhere in the room, so the maximum uncertainty in its horizontal position is $\Delta x = 5.0\text{ m}$.

(b) The uncertainty principle gives $\Delta x\Delta p_x \ge \hbar/2$, and $\Delta p_x = m\Delta v_x$ since we know the mosquito's mass. This gives $\Delta x\, m\Delta v_x \ge \hbar/2$, which we can solve for Δv_x to get the minimum uncertainty in v_x.

$\Delta v_x = \dfrac{\hbar}{2m\Delta x} = \dfrac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2(1.5\times10^{-6}\text{ kg})(5.0\text{ m})} = 7.0\times10^{-30}\text{ m/s}$, which is hardly a serious impediment!

EVALUATE: For something as "large" as a mosquito, the uncertainty principle places a negligible limitation on our ability to measure its speed.

39.49. **(a) IDENTIFY** and **SET UP:** Use $\Delta x\Delta p_x \ge \hbar/2$ to calculate Δp_x and obtain Δv_x from this.

EXECUTE: $\Delta p_x \ge \dfrac{\hbar}{2\Delta x} = \dfrac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2(1.00\times10^{-6}\text{ m})} = 5.725\times10^{-29}\text{ kg}\cdot\text{m/s}$.

$\Delta v_x = \dfrac{\Delta p_x}{m} = \dfrac{5.275\times10^{-29}\text{ kg}\cdot\text{m/s}}{1200\text{ kg}} = 4.40\times10^{-32}\text{ m/s}$.

(b) EVALUATE: Even for this very small Δx the minimum Δv_x required by the Heisenberg uncertainty principle is very small. The uncertainty principle does not impose any practical limit on the simultaneous measurements of the positions and velocities of ordinary objects.

39.50. **IDENTIFY:** Since we know that the marble is somewhere on the table, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is therefore an uncertainty in its momentum.

SET UP: The uncertainty principle is $\Delta x \Delta p_x \ge \hbar/2$.

EXECUTE: **(a)** Since the marble is somewhere on the table, the maximum uncertainty in its horizontal position is $\Delta x = 1.75$ m.

(b) Following the same procedure as in part (b) of Problem 39.48, the minimum uncertainty in the horizontal velocity of the marble is $\Delta v_x = \dfrac{\hbar}{2m\Delta x} = \dfrac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2(0.0100\text{ kg})(1.75\text{ m})} = 3.01\times10^{-33}$ m/s.

(c) The uncertainty principle tells us that we cannot know that the marble's horizontal velocity is *exactly* zero, so the smallest we could measure it to be is 3.01×10^{-33} m/s, from part (b). The longest time it could remain on the table is the time to travel the full width of the table (1.75 m), so $t = x/v_x = (1.75\text{ m})/(3.01\times10^{-33}\text{ m/s}) = 5.81\times10^{32}\text{ s} = 1.84\times10^{25}$ years. Since the universe is about 14×10^9 years old, this time is about $\dfrac{1.8\times10^{25}\text{ yr}}{14\times10^9\text{ yr}} \approx 1.3\times10^{15}$ times the age of the universe! Don't hold your breath!

EVALUATE: For household objects, the uncertainty principle places a negligible limitation on our ability to measure their speed.

39.51. **IDENTIFY:** Heisenberg's Uncertainty Principles tells us that $\Delta x \Delta p_x \ge \hbar/2$.

SET UP: We can treat the standard deviation as a direct measure of uncertainty.

EXECUTE: Here $\Delta x \Delta p_x = (1.2\times10^{-10}\text{ m})(3.0\times10^{-25}\text{ kg}\cdot\text{m/s}) = 3.6\times10^{-35}\text{ J}\cdot\text{s}$, but $\hbar/2 = 5.28\times10^{-35}\text{ J}\cdot\text{s}$. Therefore $\Delta x \Delta p_x < \hbar/2$, so the claim is *not valid*.

EVALUATE: The uncertainty product $\Delta x \Delta p_x$ must increase by a factor of about 1.5 to become consistent with the Heisenberg Uncertainty Principle.

39.52. **IDENTIFY:** Apply the Heisenberg Uncertainty Principle.

SET UP: $\Delta p_x = m\Delta v_x$.

EXECUTE: **(a)** $(\Delta x)(m\Delta v_x) \ge \hbar/2$, and setting $\Delta v_x = (0.010)v_x$ and the product of the uncertainties equal to $\hbar/2$ (for the minimum uncertainty) gives $v_x = \hbar/[2m(0.010)\Delta x] = 29.0$ m/s.

(b) Repeating with the proton mass gives 15.8 mm/s.

EVALUATE: For a given Δp_x, Δv_x is smaller for a proton than for an electron, since the proton has larger mass.

39.53. **IDENTIFY:** Apply the Heisenberg Uncertainty Principle in the form $\Delta E \Delta t = \hbar/2$.

SET UP: Let $\Delta t = 5.2\times10^{-3}$ s, the lifetime of the state of the atom, and let ΔE be the uncertainty in the energy of the state.

EXECUTE: $\Delta E > \dfrac{\hbar}{2\Delta t} = \dfrac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{2(5.2\times10^{-3}\text{ s})} = 1.01\times10^{-32}\text{ J} = 6.34\times10^{-14}\text{ eV}$.

EVALUATE: The uncertainty in the energy is a very small fraction of the typical energy of atomic states, which is on the order of 1 eV.

39.54. **IDENTIFY** and **SET UP:** The Heisenberg Uncertainty Principle says $\Delta x \Delta p_x \ge \hbar/2$. The minimum allowed $\Delta x \Delta p_x$ is $\hbar/2$. $\Delta p_x = m\Delta v_x$.

EXECUTE: **(a)** $m\Delta x \Delta v_x = \hbar/2$. $\Delta v_x = \dfrac{\hbar}{2m\Delta x} = \dfrac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2(1.67\times10^{-27}\text{ kg})(2.0\times10^{-12}\text{ m})} = 1.6\times10^4$ m/s.

(b) $\Delta x = \dfrac{\hbar}{2m\Delta v_x} = \dfrac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2(9.11\times10^{-31}\text{ kg})(0.250\text{ m/s})} = 2.3\times10^{-4}$ m.

EVALUATE: The smaller Δx is, the larger Δv_x must be.

39.55. **(a) IDENTIFY** and **SET UP:** Apply Eq. (39.17): $m_r = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{207 m_e m_p}{207 m_e + m_p}$

EXECUTE: $m_r = \dfrac{207(9.109\times10^{-31}\text{ kg})(1.673\times10^{-27}\text{ kg})}{207(9.109\times10^{-31}\text{ kg}) + 1.673\times10^{-27}\text{ kg}} = 1.69\times10^{-28}\text{ kg}$

We have used m_e to denote the electron mass.

(b) IDENTIFY: In Eq. (39.14) replace $m = m_e$ by m_r: $E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{m_r e^4}{8n^2h^2}$.

SET UP: Write as $E_n = \left(\dfrac{m_r}{m_H}\right)\left(-\dfrac{1}{\epsilon_0^2}\dfrac{m_H e^4}{8n^2h^2}\right)$, since we know that $\dfrac{1}{\epsilon_0^2}\dfrac{m_H e^4}{8h^2} = 13.60\text{ eV}$. Here m_H denotes the reduced mass for the hydrogen atom; $m_H = 0.99946(9.109\times10^{-31}\text{ kg}) = 9.104\times10^{-31}\text{ kg}$.

EXECUTE: $E_n = \left(\dfrac{m_r}{m_H}\right)\left(-\dfrac{13.60\text{ eV}}{n^2}\right)$

$$E_1 = \frac{1.69\times10^{-28}\text{ kg}}{9.109\times10^{-31}\text{ kg}}(-13.60\text{ eV}) = 186(-13.60\text{ eV}) = -2.53\text{ keV}$$

(c) SET UP: From part (b), $E_n = \left(\dfrac{m_r}{m_H}\right)\left(-\dfrac{R_H ch}{n^2}\right)$, where $R_H = 1.097\times10^7\text{ m}^{-1}$ is the Rydberg constant for the hydrogen atom. Use this result in $\dfrac{hc}{\lambda} = E_i - E_f$ to find an expression for $1/\lambda$. The initial level for the transition is the $n_i = 2$ level and the final level is the $n_f = 1$ level.

EXECUTE: $\dfrac{hc}{\lambda} = \dfrac{m_r}{m_H}\left(-\dfrac{R_H ch}{n_i^2} - \left(-\dfrac{R_H ch}{n_f^2}\right)\right)$

$$\frac{1}{\lambda} = \frac{m_r}{m_H}R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{\lambda} = \frac{1.69\times10^{-28}\text{ kg}}{9.109\times10^{-31}\text{ kg}}(1.097\times10^7\text{ m}^{-1})\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 1.527\times10^9\text{ m}^{-1}$$

$\lambda = 0.655\text{ nm}$

EVALUATE: From Example 39.6 the wavelength of the radiation emitted in this transition in hydrogen is 122 nm. The wavelength for muonium is $\dfrac{m_H}{m_r} = 5.39\times10^{-3}$ times this. The reduced mass for hydrogen is very close to the electron mass because the electron mass is much less then the proton mass: $m_p/m_e = 1836$. The muon mass is $207m_e = 1.886\times10^{-28}$ kg. The proton is only about 10 times more massive than the muon, so the reduced mass is somewhat smaller than the muon mass. The muon-proton atom has much more strongly bound energy levels and much shorter wavelengths in its spectrum than for hydrogen.

39.56. **IDENTIFY:** Apply conservation of momentum to the system of atom and emitted photon.

SET UP: Assume the atom is initially at rest. For a photon $E = \dfrac{hc}{\lambda}$ and $p = \dfrac{h}{\lambda}$.

EXECUTE: **(a)** Assume a non-relativistic velocity and conserve momentum $\Rightarrow mv = \dfrac{h}{\lambda} \Rightarrow v = \dfrac{h}{m\lambda}$.

(b) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\left(\dfrac{h}{m\lambda}\right)^2 = \dfrac{h^2}{2m\lambda^2}$.

(c) $\dfrac{K}{E}=\dfrac{h^2}{2m\lambda^2}\cdot\dfrac{\lambda}{hc}=\dfrac{h}{2mc\lambda}$. Recoil becomes an important concern for small m and small λ since this ratio becomes large in those limits.

(d) $E=10.2\text{ eV}\Rightarrow\lambda=\dfrac{hc}{E}=\dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(10.2\text{ eV})(1.60\times10^{-19}\text{ J/eV})}=1.22\times10^{-7}\text{ m}=122\text{ nm}.$

$$K=\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{2(1.67\times10^{-27}\text{ kg})(1.22\times10^{-7}\text{ m})^2}=8.84\times10^{-27}\text{ J}=5.53\times10^{-8}\text{ eV}.$$

$\dfrac{K}{E}=\dfrac{5.53\times10^{-8}\text{ eV}}{10.2\text{ eV}}=5.42\times10^{-9}$. This is quite small so recoil can be neglected.

EVALUATE: For emission of photons with ultraviolet or longer wavelengths the recoil kinetic energy of the atom is much less than the energy of the emitted photon.

39.57. **IDENTIFY** and **SET UP:** The H_α line in the Balmer series corresponds to the $n=3$ to $n=2$ transition.

$E_n=-\dfrac{13.6\text{ eV}}{n^2}$. $\dfrac{hc}{\lambda}=\Delta E.$

EXECUTE: **(a)** The atom must be given an amount of energy $E_3-E_1=-(13.6\text{ eV})\left(\dfrac{1}{3^2}-\dfrac{1}{1^2}\right)=12.1\text{ eV}.$

(b) There are three possible transitions. $n=3\to n=1$: $\Delta E=12.1\text{ eV}$ and $\lambda=\dfrac{hc}{\Delta E}=103\text{ nm}$;

$n=3\to n=2$: $\Delta E=-(13.6\text{ eV})\left(\dfrac{1}{3^2}-\dfrac{1}{2^2}\right)=1.89\text{ eV}$ and $\lambda=657\text{ nm}$; $n=2\to n=1$:

$\Delta E=-(13.6\text{ eV})\left(\dfrac{1}{2^2}-\dfrac{1}{1^2}\right)=10.2\text{ eV}$ and $\lambda=122\text{ nm}.$

EVALUATE: The larger the transition energy for the atom, the shorter the wavelength.

39.58. **IDENTIFY:** Apply $\dfrac{n_2}{n_1}=e^{-(E_{\text{ex}}-E_{\text{g}})/kT}$.

SET UP: $E_{\text{ex}}=E_2=\dfrac{-13.6\text{ eV}}{4}=-3.4\text{ eV}$. $E_{\text{g}}=-13.6\text{ eV}$. $E_{\text{ex}}-E_{\text{g}}=10.2\text{ eV}=1.63\times10^{-18}\text{ J}.$

EXECUTE: **(a)** $T=\dfrac{-(E_{\text{ex}}-E_{\text{g}})}{k\ln(n_2/n_1)}$. $\dfrac{n_2}{n_1}=10^{-12}$. $T=\dfrac{-(1.63\times10^{-18}\text{ J})}{(1.38\times10^{-23}\text{ J/K})\ln(10^{-12})}=4275\text{ K}.$

(b) $\dfrac{n_2}{n_1}=10^{-8}$. $T=\dfrac{-(1.63\times10^{-18}\text{ J})}{(1.38\times10^{-23}\text{ J/K})\ln(10^{-8})}=6412\text{ K}.$

(c) $\dfrac{n_2}{n_1}=10^{-4}$. $T=\dfrac{-(1.63\times10^{-18}\text{ J})}{(1.38\times10^{-23}\text{ J/K})\ln(10^{-4})}=12824\text{ K}.$

EVALUATE: **(d)** For absorption to take place in the Balmer series, hydrogen must *start* in the $n=2$ state. From part (a), colder stars have fewer atoms in this state leading to weaker absorption lines.

39.59. **(a) IDENTIFY** and **SET UP:** The photon energy is given to the electron in the atom. Some of this energy overcomes the binding energy of the atom and what is left appears as kinetic energy of the free electron. Apply $hf=E_{\text{f}}-E_{\text{i}}$, the energy given to the electron in the atom when a photon is absorbed.

EXECUTE: The energy of one photon is $\dfrac{hc}{\lambda}=\dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{85.5\times10^{-9}\text{ m}}$

$\dfrac{hc}{\lambda}=2.323\times10^{-18}\text{ J}(1\text{ eV}/1.602\times10^{-19}\text{ J})=14.50\text{ eV}.$

The final energy of the electron is $E_{\text{f}}=E_{\text{i}}+hf$. In the ground state of the hydrogen atom the energy of the electron is $E_{\text{i}}=-13.60\text{ eV}$. Thus $E_{\text{f}}=-13.60\text{ eV}+14.50\text{ eV}=0.90\text{ eV}.$

(b) EVALUATE: At thermal equilibrium a few atoms will be in the $n = 2$ excited levels, which have an energy of $-13.6\text{ eV}/4 = -3.40\text{ eV}, 10.2\text{ eV}$ greater than the energy of the ground state. If an electron with $E = -3.40\text{ eV}$ gains 14.5 eV from the absorbed photon, it will end up with $14.5\text{ eV} - 3.4\text{ eV} = 11.1\text{ eV}$ of kinetic energy.

39.60. IDENTIFY: For circular motion, $L = mvr$ and $a = \dfrac{v^2}{r}$. Newton's law of gravitation is $F_g = G\dfrac{mM}{r^2}$, with $G = 6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2$.

SET UP: The period T is $2.00\text{ h} = 7200\text{ s}$.

EXECUTE: **(a)** $mvr = n\dfrac{h}{2\pi}$. $n = \dfrac{2\pi mvr}{h}$. $v = \dfrac{2\pi r}{T}$ So

$$n = \frac{(2\pi r)^2 m}{hT} = \frac{(2\pi)^2(8.06\times10^6\text{ m})^2(20.0\text{ kg})}{(6.63\times10^{-34}\text{ J.s})(7200\text{ s})} = 1.08\times10^{46}.$$

(b) $F = ma$ gives $G\dfrac{mm_E}{r^2} = m\dfrac{v^2}{r}$. $\dfrac{Gm_E}{r} = v^2$. The Bohr postulate says $v = \dfrac{nh}{2\pi mr}$ so $\dfrac{Gm_E}{r} = \dfrac{n^2h^2}{4\pi^2m^2r^2}$

$$r = \left(\frac{h^2}{4\pi^2Gm_Em^2}\right)n^2.$$ This is in the form $r = kn^2$, with

$$k = \frac{h^2}{4\pi^2Gm_Em^2} = \frac{(6.63\times10^{-34}\text{ J.s})^2}{4\pi^2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})^2} = 7.0\times10^{-86}\text{ m}$$

(c) $\Delta r = r_{n+1} - r_n = k([n+1]^2 - n^2) = (2n+1)k = (2[1.08\times10^{46}]+1)(7.0\times10^{-86}\text{ m}) = 1.5\times10^{-39}\text{ m}$

EVALUATE: **(d)** Δr is exceedingly small, so the separation of adjacent orbits is not observable.

(e) There is no measurable difference between quantized and classical orbits for this satellite; either method of calculation is totally acceptable.

39.61. IDENTIFY: Assuming that Betelgeuse radiates like a perfect blackbody, Wien's displacement and the Stefan-Boltzmann law apply to its radiation.

SET UP: Wien's displacement law is $\lambda_{\text{peak}} = \dfrac{2.90\times10^{-3}\text{ m}\cdot\text{K}}{T}$, and the Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma AT^4$.

EXECUTE: **(a)** First use Wien's law to find the peak wavelength:

$$\lambda_m = (2.90\times10^{-3}\text{ m}\cdot\text{K})/(3000\text{ K}) = 9.667\times10^{-7}\text{ m}$$

Call N the number of photons/second radiated. $N\times(\text{energy per photon}) = IA = \sigma AT^4$.

$N(hc/\lambda_m) = \sigma AT^4$. $N = \dfrac{\lambda_m\sigma AT^4}{hc}$.

$$N = \frac{(9.667\times10^{-7}\text{ m})(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(4\pi)(600\times6.96\times10^8\text{ m})^2(3000\text{ K})^4}{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}.$$

$N = 5\times10^{49}$ photons/s.

(b) $$\frac{I_BA_B}{I_SA_S} = \frac{\sigma A_BT_B^4}{\sigma A_ST_S^4} = \frac{4\pi R_B^2T_B^4}{4\pi R_S^2T_S^4} = \left(\frac{600R_S}{R_S}\right)^2\left(\frac{3000\text{ K}}{5800\text{ K}}\right)^4 = 3\times10^4$$

EVALUATE: Betelgeuse radiates 30,000 times as much energy per second as does our sun!

39.62. **IDENTIFY:** The diffraction grating allows us to determine the peak-intensity wavelength of the light. Then Wien's displacement law allows us to calculate the temperature of the blackbody, and the Stefan-Boltzmann law allows us to calculate the rate at which it radiates energy.

SET UP: The bright spots for a diffraction grating occur when $d\sin\theta = m\lambda$. Wien's displacement law is $\lambda_{\text{peak}} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{T}$, and the Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: **(a)** First find the wavelength of the light:

$$\lambda = d\sin\theta = [1/(385{,}000\ \text{lines/m})]\sin(11.6°) = 5.22\times10^{-7}\ \text{m}$$

Now use Wien's law to find the temperature: $T = (2.90\times10^{-3}\ \text{m}\cdot\text{K})/(5.22\times10^{-7}\ \text{m}) = 5550\ \text{K}$.

(b) The energy radiated by the blackbody is equal to the power times the time, giving $U = Pt = IAt = \sigma A T^4 t$, which gives

$t = U/(\sigma A T^4) = (12.0\times10^6\ \text{J})/[(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(4\pi)(0.0750\ \text{m})^2(5550\ \text{K})^4] = 3.16\ \text{s}$.

EVALUATE: By ordinary standards, this blackbody is very hot, so it does not take long to radiate 12.0 MJ of energy.

39.63. **IDENTIFY:** The energy of the peak-intensity photons must be equal to the energy difference between the $n=1$ and the $n=4$ states. Wien's law allows us to calculate what the temperature of the blackbody must be for it to radiate with its peak intensity at this wavelength.

SET UP: In the Bohr model, the energy of an electron in shell n is $E_n = -\dfrac{13.6\ \text{eV}}{n^2}$, and Wien's displacement law is $\lambda_{\text{m}} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{T}$. The energy of a photon is $E = hf = hc/\lambda$.

EXECUTE: First find the energy (ΔE) that a photon would need to excite the atom. The ground state of the atom is $n=1$ and the third excited state is $n=4$. This energy is the *difference* between the two energy levels. Therefore $\Delta E = (-13.6\ \text{eV})\left(\dfrac{1}{4^2} - \dfrac{1}{1^2}\right) = 12.8\ \text{eV}$. Now find the wavelength of the photon having this amount of energy. $hc/\lambda = 12.8\ \text{eV}$ and

$$\lambda = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^8\ \text{m/s})/(12.8\ \text{eV}) = 9.73\times10^{-8}\ \text{m}$$

Now use Wien's law to find the temperature. $T = (0.00290\ \text{m}\cdot\text{K})/(9.73\times10^{-8}\ \text{m}) = 2.98\times10^4\ \text{K}$.

EVALUATE: This temperature is well above ordinary room temperatures, which is why hydrogen atoms are not in excited states during everyday conditions.

39.64. **IDENTIFY:** The blackbody radiates heat into the water, but the water also radiates heat back into the blackbody. The net heat entering the water causes evaporation. Wien's law tells us the peak wavelength radiated, but a thermophile in the water measures the wavelength and frequency of the light in the water.

SET UP: By the Stefan-Boltzman law, the net power radiated by the blackbody is $\dfrac{dQ}{dt} = \sigma A\left(T^4_{\text{sphere}} - T^4_{\text{water}}\right)$. Since this heat evaporates water, the rate at which water evaporates is $\dfrac{dQ}{dt} = L_{\text{v}}\dfrac{dm}{dt}$. Wien's displacement law is $\lambda_{\text{m}} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{T}$, and the wavelength in the water is $\lambda_{\text{w}} = \lambda_0/n$.

EXECUTE: **(a)** The net radiated heat is $\dfrac{dQ}{dt} = \sigma A\left(T^4_{\text{sphere}} - T^4_{\text{water}}\right)$ and the evaporation rate is $\dfrac{dQ}{dt} = L_{\text{v}}\dfrac{dm}{dt}$, where *dm* is the mass of water that evaporates in time *dt*. Equating these two rates gives

$L_v \dfrac{dm}{dt} = \sigma A\left(T_{\text{sphere}}^4 - T_{\text{water}}^4\right)$. $\dfrac{dm}{dt} = \dfrac{\sigma(4\pi R^2)\left(T_{\text{sphere}}^4 - T_{\text{water}}^4\right)}{L_v}$.

$$\frac{dm}{dt} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(4\pi)(0.120\ \text{m})^2\left[(498\ \text{K})^4 - (373\ \text{K})^4\right]}{2256\times10^3\ \text{J/kg}} = 1.92\times10^{-4}\ \text{kg/s} = 0.193\ \text{g/s}$$

(b) (i) Wien's law gives $\lambda_m = (0.00290\ \text{m}\cdot\text{K})/(498\ \text{K}) = 5.82\times10^{-6}$ m

But this would be the wavelength in vacuum. In the water the thermophile organism would measure $\lambda_w = \lambda_0/n = (5.82\times10^{-6}\ \text{m})/1.333 = 4.37\times10^{-6}\ \text{m} = 4.37\ \mu\text{m}$

(ii) The frequency is the same as if the wave were in air, so

$$f = c/\lambda_0 = (3.00\times10^8\ \text{m/s})/(5.82\times10^{-6}\ \text{m}) = 5.15\times10^{13}\ \text{Hz}$$

EVALUATE: An alternative way is to use the quantities in the water: $f = \dfrac{c/n}{\lambda_0/n} = c/\lambda_0$, which gives the same answer for the frequency. An organism in the water would measure the light coming to it through the water, so the wavelength it would measure would be reduced by a factor of $1/n$.

39.65. **IDENTIFY:** Apply conservation of energy and conservation of linear momentum to the system of atom plus photon.

(a) SET UP: Let E_{tr} be the transition energy, E_{ph} be the energy of the photon with wavelength λ', and E_{r} be the kinetic energy of the recoiling atom. Conservation of energy gives $E_{\text{ph}} + E_{\text{r}} = E_{\text{tr}}$.

$E_{\text{ph}} = \dfrac{hc}{\lambda'}$ so $\dfrac{hc}{\lambda'} = E_{\text{tr}} - E_{\text{r}}$ and $\lambda' = \dfrac{hc}{E_{\text{tr}} - E_{\text{r}}}$.

EXECUTE: If the recoil energy is neglected then the photon wavelength is $\lambda = hc/E_{\text{tr}}$.

$$\Delta\lambda = \lambda' - \lambda = hc\left(\frac{1}{E_{\text{tr}} - E_{\text{r}}} - \frac{1}{E_{\text{tr}}}\right) = \left(\frac{hc}{E_{\text{tr}}}\right)\left(\frac{1}{1 - E_{\text{r}}/E_{\text{tr}}} - 1\right)$$

$$\frac{1}{1 - E_{\text{r}}/E_{\text{tr}}} = \left(1 - \frac{E_{\text{r}}}{E_{\text{tr}}}\right)^{-1} \approx 1 + \frac{E_{\text{r}}}{E_{\text{tr}}} \text{ since } \frac{E_{\text{r}}}{E_{\text{tr}}} \ll 1$$

(We have used the binomial theorem, Appendix B.)

Thus $\Delta\lambda = \dfrac{hc}{E_{\text{tr}}}\left(\dfrac{E_{\text{r}}}{E_{\text{tr}}}\right)$, or since $E_{\text{tr}} = hc/\lambda$, $\Delta\lambda = \left(\dfrac{E_{\text{r}}}{hc}\right)\lambda^2$.

SET UP: Use conservation of linear momentum to find E_{r}: Assuming that the atom is initially at rest, the momentum p_{r} of the recoiling atom must be equal in magnitude and opposite in direction to the momentum $p_{\text{ph}} = h/\lambda$ of the emitted photon: $h/\lambda = p_{\text{r}}$.

EXECUTE: $E_{\text{r}} = \dfrac{p_{\text{r}}^2}{2m}$, where m is the mass of the atom, so $E_{\text{r}} = \dfrac{h^2}{2m\lambda^2}$.

Use this result in the above equation: $\Delta\lambda = \left(\dfrac{E_{\text{r}}}{hc}\right)\lambda^2 = \left(\dfrac{h^2}{2m\lambda^2}\right)\left(\dfrac{\lambda^2}{hc}\right) = \dfrac{h}{2mc}$;

note that this result for $\Delta\lambda$ is independent of the atomic transition energy.

(b) For a hydrogen atom $m = m_{\text{p}}$ and $\Delta\lambda = \dfrac{h}{2m_{\text{p}}c} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{2(1.673\times10^{-27}\ \text{kg})(2.998\times10^8\ \text{m/s})} = 6.61\times10^{-16}$ m

EVALUATE: The correction is independent of n. The wavelengths of photons emitted in hydrogen atom transitions are on the order of $100\ \text{nm} = 10^{-7}$ m, so the recoil correction is exceedingly small.

39.66. **IDENTIFY:** Combine $I = \sigma T^4$, $P = IA$, and $\Delta E = Pt$.

SET UP: In the Stefan-Boltzmann law the temperature must be in kelvins. 200°C = 473 K.

EXECUTE: $t = \dfrac{\Delta E}{A\sigma T^4} = \dfrac{(100\text{ J})}{(4.00\times 10^{-6}\text{ m}^2)(5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)(473\text{ K})^4} = 8.81\times 10^3\text{ s} = 2.45\text{ h}.$

EVALUATE: $P = 0.0114$ W. Since the area of the hole is small, the rate at which the cavity radiates energy through the hole is very small.

39.67. **IDENTIFY** and **SET UP:** Follow the procedures specified in the problem.

EXECUTE: **(a)** $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$ but $\lambda = \dfrac{c}{f} \Rightarrow I(f) = \dfrac{2\pi hc^2}{(c/f)^5(e^{hf/kT}-1)} = \dfrac{2\pi hf^5}{c^3(e^{hf/kT}-1)}$

(b) $\int_0^\infty I(\lambda)\,d\lambda = \int_\infty^0 I(f)\,df\left(\dfrac{-c}{f^2}\right)$

$$= \int_0^\infty \frac{2\pi hf^3\,df}{c^2(e^{hf/kT}-1)} = \frac{2\pi(kT)^4}{c^2h^3}\int_0^\infty \frac{x^3}{e^x-1}\,dx = \frac{2\pi(kT)^4}{c^2h^3}\frac{1}{240}(2\pi)^4 = \frac{(2\pi)^5(kT)^4}{240h^3c^2} = \frac{2\pi^5k^4T^4}{15c^2h^3}.$$

(c) The expression $\dfrac{2\pi^5k^4}{15h^3c^2} = \sigma$ as shown in Eq. (39.28). Plugging in the values for the constants we get $\sigma = 5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4$.

EVALUATE: The Planck radiation law, Eq. (39.24), predicts the Stefan-Boltzmann law, Eq. (39.19).

39.68. **IDENTIFY:** $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$. From Chapter 36, if $\lambda \ll a$ then the width w of the central maximum is $w = 2\dfrac{R\lambda}{a}$, where $R = 2.5$ m and a is the width of the slit.

SET UP: $v_x = \sqrt{\dfrac{2E}{m}}$, since the beam is traveling in the x-direction and $\Delta v_y \ll v_x$

EXECUTE: **(a)** $\lambda = \dfrac{h}{\sqrt{2mK}} = \dfrac{(6.63\times 10^{-34}\text{ J}\cdot\text{s})}{\sqrt{2(9.11\times 10^{-31}\text{ kg})(40\text{ eV})(1.60\times 10^{-19}\text{ J/eV})}} = 1.94\times 10^{-10}\text{ m}.$

(b) $\dfrac{R}{v} = \dfrac{R}{\sqrt{2E/m}} = \dfrac{(2.5\text{ m})(9.11\times 10^{-31}\text{ kg})^{1/2}}{\sqrt{2(40\text{ eV})(1.6\times 10^{-19}\text{ J/eV})}} = 6.67\times 10^{-7}\text{ s}.$

(c) The width w is $w = 2R\dfrac{\lambda}{a}$, and $w = \Delta v_y t = \Delta p_y t/m$, where t is the time found in part (b) and a is the slit width. Combining the expressions for w, $\Delta p_y = \dfrac{2m\lambda R}{at} = 2.65\times 10^{-28}\text{ kg}\cdot\text{m/s}$.

(d) $\Delta y = \dfrac{\hbar}{2\Delta p_y} = 0.20\ \mu\text{m}$, which is the same order of magnitude of the width of the slit.

EVALUATE: For these electrons $\lambda = 1.94\times 10^{-10}$ m. This is much smaller than a and the approximate expression $w = \dfrac{2R\lambda}{a}$ is very accurate. Also, $v_x = \sqrt{\dfrac{2E}{m}} = 3.75\times 10^6$ m/s. $\Delta v_y = \dfrac{\Delta p_y}{m} = 2.9\times 10^2$ m/s, so it is the case that $v_x \gg \Delta v_y$.

39.69. **IDENTIFY:** For a photon $E = \dfrac{hc}{\lambda}$. For a particle with mass, $p = \dfrac{h}{\lambda}$ and $E = \dfrac{p^2}{2m} = q\Delta V$, where ΔV is the accelerating voltage. To exhibit wave nature when passing through an opening, the de Broglie wavelength of the particle must be comparable with the width of the opening.

SET UP: An electron has mass 9.109×10^{-31} kg. A proton has mass 1.673×10^{-27} kg.

EXECUTE: **(a)** $E = hc/\lambda = 12$ eV

(b) Find E for an electron with $\lambda = 0.10\times10^{-6}$ m. $\lambda = h/p$ so $p = h/\lambda = 6.626\times10^{-27}\ \text{kg}\cdot\text{m/s}$.

$E = p^2/(2m) = 1.5\times10^{-4}$ eV. $E = q\Delta V$ so $\Delta V = 1.5\times10^{-4}$ V.

$v = p/m = (6.626\times10^{-27}\ \text{kg}\cdot\text{m/s})/(9.109\times10^{-31}\ \text{kg}) = 7.3\times10^{3}$ m/s

(c) Same λ so same p. $E = p^2/(2m)$ but now $m = 1.673\times10^{-27}$ kg so $E = 8.2\times10^{-8}$ eV and

$\Delta V = 8.2\times10^{-8}$ V. $v = p/m = (6.626\times10^{-27}\ \text{kg}\cdot\text{m/s})/(1.673\times10^{-27}\ \text{kg}) = 4.0$ m/s

EVALUATE: A proton must be traveling much slower than an electron in order to have the same de Broglie wavelength.

39.70. **IDENTIFY:** The de Broglie wavelength of the electrons must be such that the first diffraction minimum occurs at $\theta = 20.0°$.

SET UP: The single-slit diffraction minima occur at angles θ given by $a\sin\theta = m\lambda$. $p = \dfrac{h}{\lambda}$.

EXECUTE: **(a)** $\lambda = a\sin\theta = (150\times10^{-9}\ \text{m})\sin 20° = 5.13\times10^{-8}$ m. $\lambda = h/mv \to v = h/m\lambda$.

$$v = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(5.13\times10^{-8}\ \text{m})} = 1.42\times10^{4}\ \text{m/s}.$$

(b) No electrons strike the screen at the location of the second diffraction minimum. $a\sin\theta_2 = 2\lambda$.

$$\sin\theta_2 = \pm2\frac{\lambda}{a} = \pm2\left(\frac{5.13\times10^{-8}\ \text{m}}{150\times10^{-9}\ \text{m}}\right) = \pm0.684.\ \theta_2 = \pm43.2°.$$

EVALUATE: The intensity distribution in the diffraction pattern depends on the wavelength λ and is the same for light of wavelength λ as for electrons with de Broglie wavelength λ.

39.71. **IDENTIFY:** The electrons behave like waves and produce a double-slit interference pattern after passing through the slits.

SET UP: The first angle at which destructive interference occurs is given by $d\sin\theta = \lambda/2$. The de Broglie wavelength of each of the electrons is $\lambda = h/mv$.

EXECUTE: **(a)** First find the wavelength of the electrons. For the first dark fringe, we have $d\sin\theta = \lambda/2$, which gives $(1.25\ \text{nm})(\sin 18.0°) = \lambda/2$, and $\lambda = 0.7725$ nm. Now solve the de Broglie wavelength equation for the speed of the electron:

$$v = \frac{h}{m\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(0.7725\times10^{-9}\ \text{m})} = 9.42\times10^{5}\ \text{m/s}$$

which is about 0.3% the speed of light, so they are *nonrelativistic*.

(b) Energy conservation gives $eV = \frac{1}{2}mv^2$ and

$$V = mv^2/2e = (9.11\times10^{-31}\ \text{kg})(9.42\times10^{5}\ \text{m/s})^2/[2(1.60\times10^{-19}\ \text{C})] = 2.52\ \text{V}$$

EVALUATE: The hole must be much smaller than the wavelength of visible light for the electrons to show diffraction.

39.72. **IDENTIFY:** The alpha particles and protons behave as waves and exhibit circular-aperture diffraction after passing through the hole.

SET UP: For a round hole, the first dark ring occurs at the angle θ for which $\sin\theta = 1.22\lambda/D$, where D is the diameter of the hole. The de Broglie wavelength for a particle is $\lambda = h/p = h/mv$.

EXECUTE: Taking the ratio of the sines for the alpha particle and proton gives

$$\frac{\sin\theta_\alpha}{\sin\theta_\text{p}} = \frac{1.22\lambda_\alpha}{1.22\lambda_\text{p}} = \frac{\lambda_\alpha}{\lambda_\text{p}}$$

The de Broglie wavelength gives $\lambda_\text{p} = h/p_\text{p}$ and $\lambda_\alpha = h/p_\alpha$, so $\dfrac{\sin\theta_\alpha}{\sin\theta_\text{p}} = \dfrac{h/p_\alpha}{h/p_\text{p}} = \dfrac{p_\text{p}}{p_\alpha}$. Using $K = p^2/2m$,

we have $p = \sqrt{2mK}$. Since the alpha particle has twice the charge of the proton and both are accelerated through the same potential difference, $K_\alpha = 2K_\mathrm{p}$. Therefore $p_\mathrm{p} = \sqrt{2m_\mathrm{p}K_\mathrm{p}}$ and $p_\alpha = \sqrt{2m_\alpha K_\alpha} = \sqrt{2m_\alpha(2K_\mathrm{p})} = \sqrt{4m_\alpha K_\mathrm{p}}$. Substituting these quantities into the ratio of the sines gives

$$\frac{\sin\theta_\alpha}{\sin\theta_\mathrm{p}} = \frac{p_\mathrm{p}}{p_\alpha} = \frac{\sqrt{2m_\mathrm{p}K_\mathrm{p}}}{\sqrt{4m_\alpha K_\mathrm{p}}} = \sqrt{\frac{m_\mathrm{p}}{2m_\alpha}}$$

Solving for sin θ_α gives $\sin\theta_\alpha = \sqrt{\dfrac{1.67\times10^{-27}\text{ kg}}{2(6.64\times10^{-27}\text{ kg})}}\sin 15.0°$ and $\theta_\alpha = 5.3°$.

EVALUATE: Since $\sin\theta$ is inversely proportional to the mass of the particle, the larger-mass alpha particles form their first dark ring at a smaller angle than the ring for the lighter protons.

39.73. **IDENTIFY:** Both the electrons and photons behave like waves and exhibit single-slit diffraction after passing through their respective slits.

SET UP: The energy of the photon is $E = hc/\lambda$ and the de Broglie wavelength of the electron is $\lambda = h/mv = h/p$. Destructive interference for a single slit first occurs when $a\ \sin\theta = \lambda$.

EXECUTE: **(a)** For the photon: $\lambda = hc/E$ and $a\ \sin\theta = \lambda$. Since the a and θ are the same for the photons and electrons, they must both have the same wavelength. Equating these two expressions for λ gives $a\sin\theta = hc/E$. For the electron, $\lambda = h/p = \dfrac{h}{\sqrt{2mK}}$ and $a\ \sin\theta = \lambda$. Equating these two expressions for λ gives $a\ \sin\theta = \dfrac{h}{\sqrt{2mK}}$. Equating the two expressions for $a\sin\theta$ gives $hc/E = \dfrac{h}{\sqrt{2mK}}$, which gives $E = c\sqrt{2mK} = (4.05\times10^{-7}\text{ J}^{1/2})\sqrt{K}$.

(b) $\dfrac{E}{K} = \dfrac{c\sqrt{2mK}}{K} = \sqrt{\dfrac{2mc^2}{K}}$. Since $v \ll c$, $mc^2 > K$, so the square root is >1. Therefore $E/K > 1$, meaning that the photon has more energy than the electron.

EVALUATE: When a photon and a particle have the same wavelength, the photon has more energy than the particle.

39.74. **IDENTIFY:** The de Broglie wavelength of the electrons must equal the wavelength of the light.

SET UP: The maxima in the two-slit interference pattern are located by $d\sin\theta = m\lambda$. For an electron, $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$.

EXECUTE: $\lambda = \dfrac{d\sin\theta}{m} = \dfrac{(40.0\times10^{-6}\text{ m})\sin(0.0300\text{ rad})}{2} = 600\text{ nm}$. The velocity of an electron with this wavelength is given by Eq. (39.1). $v = \dfrac{p}{m} = \dfrac{h}{m\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(9.11\times10^{-31}\text{ kg})(600\times10^{-9}\text{ m})} = 1.21\times10^3\text{ m/s}$. Since this velocity is much smaller than c we can calculate the energy of the electron classically $K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\text{ kg})(1.21\times10^3\text{ m/s})^2 = 6.70\times10^{-25}\text{ J} = 4.19\ \mu\text{eV}$.

EVALUATE: The energy of the photons of this wavelength is $E = \dfrac{hc}{\lambda} = 2.07\text{ eV}$. The photons and electrons have the same wavelength but very different energies.

39.75. **IDENTIFY** and **SET UP:** The de Broglie wavelength of the blood cell is $\lambda = \dfrac{h}{mv}$.

EXECUTE: $\lambda = \dfrac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(1.00\times10^{-14}\text{ kg})(4.00\times10^{-3}\text{ m/s})} = 1.66\times10^{-17}\text{ m}$.

EVALUATE: We need not be concerned about wave behavior.

39.76. **IDENTIFY:** An electron and a photon both have the same wavelength. We want to use this fact to calculate the energy of each of them.

SET UP: The de Broglie wavelength is $\lambda = \dfrac{h}{p}$. The energy of the electron is its kinetic energy, $K = \frac{1}{2}mv^2 = p^2/2m$. The energy of the photon is $E = hf = hc/\lambda$.

EXECUTE: **(a)** $p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{400\times10^{-9}\ \text{m}} = 1.656\times10^{-27}\ \text{kg}\cdot\text{m/s}.$

$$E = \frac{p^2}{2m} = \frac{(1.656\times10^{-27}\ \text{kg}\cdot\text{m/s})^2}{2(9.109\times10^{-31}\ \text{kg})} = 1.506\times10^{-24}\ \text{J} = 9.40\times10^{-6}\ \text{eV}$$

(b) $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{400\times10^{-9}\ \text{m}} = 4.966\times10^{-19}\ \text{J} = 3.10\ \text{eV}$

EVALUATE: The photon has around 300,000 times as much energy as the electron.

39.77. **IDENTIFY** and **SET UP:** Follow the procedures specified in the problem.

EXECUTE: **(a)** $\lambda = \dfrac{h}{p} = \dfrac{h\left(1-\dfrac{v^2}{c^2}\right)^{1/2}}{mv} \Rightarrow \lambda^2 m^2 v^2 = h^2\left(1-\dfrac{v^2}{c^2}\right) = h^2 - \dfrac{h^2v^2}{c^2} \Rightarrow \lambda^2m^2v^2 + h^2\dfrac{v^2}{c^2} = h^2$

$$\Rightarrow v^2 = \frac{h^2}{\left(\lambda^2m^2 + \dfrac{h^2}{c^2}\right)} = \frac{c^2}{\left(\dfrac{\lambda^2m^2c^2}{h^2}+1\right)} \Rightarrow v = \frac{c}{\left(1+\left(\dfrac{mc\lambda}{h}\right)^2\right)^{1/2}}.$$

(b) $v = \dfrac{c}{\left(1+\left(\dfrac{\lambda}{(h/mc)}\right)^2\right)^{1/2}} \approx c\left(1-\dfrac{1}{2}\left(\dfrac{mc\lambda}{h}\right)^2\right) = (1-\Delta)c.\quad \Delta = \dfrac{m^2c^2\lambda^2}{2h^2}.$

(c) $\lambda = 1.00\times10^{-15}\text{m} \ll \dfrac{h}{mc}.\quad \Delta = \dfrac{(9.11\times10^{-31}\ \text{kg})^2(3.00\times10^{8}\ \text{m/s})^2(1.00\times10^{-15}\ \text{m})^2}{2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2} = 8.50\times10^{-8}$

$\Rightarrow v = (1-\Delta)c = (1-8.50\times10^{-8})c.$

EVALUATE: As $\Delta \to 0$, $v \to c$ and $\lambda \to 0$.

39.78. **IDENTIFY** and **SET UP:** The minimum uncertainty product is $\Delta x \Delta p_x = \hbar/2$. $\Delta x = r_1$, where r_1 is the radius of the $n=1$ Bohr orbit. In the $n=1$ Bohr orbit, $mv_1r_1 = \dfrac{h}{2\pi}$ and $p_1 = mv_1 = \dfrac{h}{2\pi r_1}$.

EXECUTE: $\Delta p_x = \dfrac{\hbar}{2\Delta x} = \dfrac{\hbar}{2r_1} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(0.529\times10^{-10}\ \text{m})} = 1.0\times10^{-24}\ \text{kg}\cdot\text{m/s}.$ This is the same as the magnitude of the momentum of the electron in the $n=1$ Bohr orbit.

EVALUATE: Since the momentum is the same order of magnitude as the uncertainty in the momentum, the uncertainty principle plays a large role in the structure of atoms.

39.79. **IDENTIFY** and **SET UP:** Combining the two equations in the hint gives $pc = \sqrt{K(K+2mc^2)}$ and $\lambda = \dfrac{hc}{\sqrt{K(K+2mc^2)}}$.

EXECUTE: **(a)** With $K = 3mc^2$ this becomes $\lambda = \dfrac{hc}{\sqrt{3mc^2(3mc^2+2mc^2)}} = \dfrac{h}{\sqrt{15}mc}.$

(b) (i) $K = 3mc^2 = 3(9.109\times10^{-31}\ \text{kg})(2.998\times10^{8}\ \text{m/s})^2 = 2.456\times10^{13}\ \text{J} = 1.53\ \text{MeV}$

$$\lambda = \frac{h}{\sqrt{15}mc} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{15}(9.109\times10^{-31}\ \text{kg})(2.998\times10^{8}\ \text{m/s})} = 6.26\times10^{-13}\ \text{m}$$

(ii) K is proportional to m, so for a proton $K = (m_p/m_e)(1.53\ \text{MeV}) = 1836(1.53\ \text{MeV}) = 2810\ \text{MeV}$

λ is proportional to $1/m$, so for a proton

$\lambda = (m_e/m_p)(6.26\times10^{-13}\ \text{m}) = (1/1836)(6.26\times10^{-13}\ \text{m}) = 3.41\times10^{-16}\ \text{m}.$

EVALUATE: The proton has a larger rest mass energy so its kinetic energy is larger when $K = 3mc^2$. The proton also has larger momentum so has a smaller λ.

39.80. **IDENTIFY:** Apply the Heisenberg Uncertainty Principle. Consider only one component of position and momentum.

SET UP: $\Delta x\Delta p_x \ge \hbar/2$. Take $\Delta x \approx 5.0\times10^{-15}$ m. $K = E - mc^2$. For a proton, $m = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\Delta p_x = \dfrac{\hbar}{2\Delta x} = \dfrac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{2(5.0\times10^{-15}\ \text{m})} = 1.1\times10^{-20}\ \text{kg}\cdot\text{m/s}.$

(b) $K = \sqrt{(pc)^2 + (mc^2)^2} - mc^2 = 3.3\times10^{-14}\ \text{J} = 0.21\ \text{MeV}.$

EVALUATE: **(c)** The result of part (b), about 2×10^5 eV, is many orders of magnitude larger than the potential energy of an electron in a hydrogen atom.

39.81. **(a) IDENTIFY** and **SET UP:** $\Delta x\Delta p_x \ge \hbar/2$. Estimate Δx as $\Delta x \approx 5.0\times10^{-15}$ m.

EXECUTE: Then the minimum allowed Δp_x is $\Delta p_x \approx \dfrac{\hbar}{2\Delta x} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(5.0\times10^{-15}\ \text{m})} = 1.1\times10^{-20}\ \text{kg}\cdot\text{m/s}.$

(b) IDENTIFY and **SET UP:** Assume $p \approx 1.1\times10^{-20}$ kg · m/s. Use Eq. (37.39) to calculate E, and then $K = E - mc^2$.

EXECUTE: $E = \sqrt{(mc^2)^2 + (pc)^2}$. $mc^2 = (9.109\times10^{-31}\ \text{kg})(2.998\times10^{8}\ \text{m/s})^2 = 8.187\times10^{-14}\ \text{J}.$

$pc = (1.1\times10^{-20}\ \text{kg}\cdot\text{m/s})(2.998\times10^{8}\ \text{m/s}) = 3.165\times10^{-12}\ \text{J}.$

$E = \sqrt{(8.187\times10^{-14}\ \text{J})^2 + (3.165\times10^{-12}\ \text{J})^2} = 3.166\times10^{-12}\ \text{J}.$

$K = E - mc^2 = 3.166\times10^{-12}\ \text{J} - 8.187\times10^{-14}\ \text{J} = 3.084\times10^{-12}\ \text{J}\times(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 19\ \text{MeV}.$

(c) IDENTIFY and **SET UP:** The Coulomb potential energy for a pair of point charges is given by Eq. (23.9). The proton has charge $+e$ and the electron has charge $-e$.

EXECUTE: $U = -\dfrac{ke^2}{r} = -\dfrac{(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\ \text{C})^2}{5.0\times10^{-15}\ \text{m}} = -4.6\times10^{-14}\ \text{J} = -0.29\ \text{MeV}.$

EVALUATE: The kinetic energy of the electron required by the uncertainty principle would be much larger than the magnitude of the negative Coulomb potential energy. The total energy of the electron would be large and positive and the electron could not be bound within the nucleus.

39.82. **IDENTIFY:** Apply the Heisenberg Uncertainty Principle. Let the uncertainty product have its minimum possible value, so $\Delta x\Delta p_x = \hbar/2$.

SET UP: Take the direction of the electron beam to be the x-direction and the direction of motion perpendicular to the beam to be the y-direction.

EXECUTE: **(a)** $\Delta v_y = \dfrac{\Delta p_y}{m} = \dfrac{\hbar}{2m\Delta y} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(9.11\times10^{-31}\ \text{kg})(0.50\times10^{-3}\ \text{m})} = 0.12\ \text{m/s}.$

(b) The uncertainty Δr in the position of the point where the electrons strike the screen is

$$\Delta r = \Delta v_y t = \frac{\Delta p_y}{m}\frac{x}{v_x} = \frac{\hbar}{2m\Delta y}\frac{x}{\sqrt{2K/m}} = 4.78\times10^{-10}\ \text{m}.$$

EVALUATE: **(c)** This is far too small to affect the clarity of the picture.

39.83. **IDENTIFY** and **SET UP:** $\Delta E \Delta t \geq \hbar/2$. Take the minimum uncertainty product, so $\Delta E = \dfrac{\hbar}{2\Delta t}$, with $\Delta t = 8.4\times 10^{-17}$ s. $m = 264m_e$. $\Delta m = \dfrac{\Delta E}{c^2}$.

EXECUTE: $\Delta E = \dfrac{1.055\times 10^{-34}\ \text{J}\cdot\text{s}}{2(8.4\times 10^{-17}\ \text{s})} = 6.28\times 10^{-19}$ J. $\Delta m = \dfrac{6.28\times 10^{-19}\ \text{J}}{(3.00\times 10^{8}\ \text{m/s})^2} = 7.0\times 10^{-36}$ kg.

$$\frac{\Delta m}{m} = \frac{7.0\times 10^{-36}\ \text{kg}}{(264)(9.11\times 10^{-31}\ \text{kg})} = 2.9\times 10^{-8}$$

EVALUATE: The fractional uncertainty in the mass is very small.

39.84. **IDENTIFY:** The insect behaves like a wave as it passes through the hole in the screen.

SET UP: **(a)** For wave behavior to show up, the wavelength of the insect must be of the order of the diameter of the hole. The de Broglie wavelength is $\lambda = h/mv$.

EXECUTE: The de Broglie wavelength of the insect must be of the order of the diameter of the hole in the screen, so $\lambda \approx 4.00$ mm. The de Broglie wavelength gives

$$v = \frac{h}{m\lambda} = \frac{6.626\times 10^{-34}\ \text{J}\cdot\text{s}}{(1.25\times 10^{-6}\ \text{kg})(0.00400\ \text{m})} = 1.33\times 10^{-25}\ \text{m/s}$$

(b) $t = x/v = (0.000500\ \text{m})/(1.33\times 10^{-25}\ \text{m/s}) = 3.77\times 10^{21}\ \text{s} = 1.4\times 10^{10}\ \text{yr}$

The universe is about 14 billion years old $(1.4\times 10^{10}\ \text{yr})$ so this time would be about 85,000 times the age of the universe.

EVALUATE: Don't expect to see a diffracting insect! Wave behavior of particles occurs only at the very small scale.

39.85. **IDENTIFY** and **SET UP:** Use Eq. (39.1) to relate your wavelength and speed.

EXECUTE: **(a)** $\lambda = \dfrac{h}{mv}$, so $v = \dfrac{h}{m\lambda} = \dfrac{6.626\times 10^{-34}\ \text{J}\cdot\text{s}}{(60.0\ \text{kg})(1.0\ \text{m})} = 1.1\times 10^{-35}$ m/s

(b) $t = \dfrac{\text{distance}}{\text{velocity}} = \dfrac{0.80\ \text{m}}{1.1\times 10^{-35}\ \text{m/s}} = 7.3\times 10^{34}\ \text{s}(1\ \text{y}/3.156\times 10^{7}\ \text{s}) = 2.3\times 10^{27}$ y

Since you walk through doorways much more quickly than this, you will not experience diffraction effects.

EVALUATE: A 1-kg object moving at 1 m/s has a de Broglie wavelength $\lambda = 6.6\times 10^{-34}$ m, which is exceedingly small. An object like you has a very, very small λ at ordinary speeds and does not exhibit wavelike properties.

39.86. **IDENTIFY:** The transition energy E for the atom and the wavelength λ of the emitted photon are related by $E = \dfrac{hc}{\lambda}$. Apply the Heisenberg Uncertainty Principle in the form $\Delta E \Delta t \geq \dfrac{\hbar}{2}$.

SET UP: Assume the minimum possible value for the uncertainty product, so that $\Delta E \Delta t = \dfrac{\hbar}{2}$.

EXECUTE: **(a)** $E = 2.58\ \text{eV} = 4.13\times 10^{-19}$ J, with a wavelength of $\lambda = \dfrac{hc}{E} = 4.82\times 10^{-7}\text{m} = 482$ nm

(b) $\Delta E = \dfrac{\hbar}{2\Delta t} = \dfrac{(1.055\times 10^{-34}\ \text{J}\cdot\text{s})}{2(1.64\times 10^{-7}\ \text{s})} = 3.22\times 10^{-28}\ \text{J} = 2.01\times 10^{-9}$ eV.

(c) $\lambda E = hc$, so $(\Delta\lambda)E + \lambda\Delta E = 0$, and $|\Delta E/E| = |\Delta\lambda/\lambda|$, so

$$\Delta\lambda = \lambda|\Delta E/E| = (4.82\times 10^{-7}\ \text{m})\left(\frac{3.22\times 10^{-28}\ \text{J}}{4.13\times 10^{-19}\ \text{J}}\right) = 3.75\times 10^{-16}\ \text{m} = 3.75\times 10^{-7}\ \text{nm}.$$

EVALUATE: The finite lifetime of the excited state gives rise to a small spread in the wavelength of the emitted light.

39.87. **IDENTIFY:** The electrons behave as waves whose wavelength is equal to the de Broglie wavelength.

SET UP: The de Broglie wavelength is $\lambda = h/mv$, and the energy of a photon is $E = hf = hc/\lambda$.

EXECUTE: **(a)** Use the de Broglie wavelength to find the speed of the electron.

$$v = \frac{h}{m\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(1.00\times10^{-9}\ \text{m})} = 7.27\times10^{5}\ \text{m/s}$$

which is much less than the speed of light, so it is nonrelativistic.

(b) Energy conservation gives $eV = \frac{1}{2}mv^2$.

$V = mv^2/2e = (9.11\times10^{-31}\ \text{kg})(7.27\times10^{5}\ \text{m/s})^2/[2(1.60\times10^{-19}\ \text{C})] = 1.51\ \text{V}$

(c) $K = eV = e(1.51\ \text{V}) = 1.51\ \text{eV}$, which is about ¼ the potential energy of the NaCl molecule, so the electron would not be too damaging.

(d) $E = hc/\lambda = (4.136\times10^{-15}\ \text{eV s})(3.00\times10^{8}\ \text{m/s})/(1.00\times10^{-9}\ \text{m}) = 1240\ \text{eV}$

which would certainly destroy the molecules under study.

EVALUATE: As we have seen in Problems 39.73 and 39.76, when a particle and a photon have the same wavelength, the photon has much more energy.

39.88. **IDENTIFY:** Assume both the x rays and electrons are at normal incidence and scatter from the surface plane of the crystal, so the maxima are located by $d\sin\theta = m\lambda$, where d is the separation between adjacent atoms in the surface plane.

SET UP: Let primed variables refer to the electrons. $\lambda' = \dfrac{h}{p'} = \dfrac{h}{\sqrt{2mE'}}$.

EXECUTE: $\sin\theta' = \dfrac{\lambda'}{\lambda}\sin\theta$, and $\lambda' = (h/p') = (h/\sqrt{2mE'})$, and so $\theta' = \arcsin\left(\dfrac{h}{\lambda\sqrt{2mE'}}\sin\theta\right)$.

$$\theta' = \arcsin\left(\frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})\sin35.8^\circ}{(3.00\times10^{-11}\ \text{m})\sqrt{2(9.11\times10^{-31}\ \text{kg})(4.50\times10^{+3}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}}\right) = 20.9^\circ.$$

EVALUATE: The x rays and electrons have different wavelengths and the $m = 1$ maxima occur at different angles.

39.89. **IDENTIFY:** The interference pattern for electrons with de Broglie wavelength λ is the same as for light with wavelength λ.

SET UP: For an electron, $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$.

EXECUTE: **(a)** The maxima occur when $2d\sin\theta = m\lambda$.

(b) $\lambda = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-37}\ \text{kg})(71.0\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}} = 1.46\times10^{-10}\,\text{m} = 0.146\ \text{nm}$. $\theta = \sin^{-1}\left(\dfrac{m\lambda}{2d}\right)$.

(Note: This m is the order of the maximum, not the mass.) $\theta = \sin^{-1}\left(\dfrac{(1)(1.46\times10^{-10}\ \text{m})}{2(9.10\times10^{-11}\ \text{m})}\right) = 53.3^\circ$.

EVALUATE: **(c)** The work function of the metal acts like an attractive potential increasing the kinetic energy of incoming electrons by $e\phi$. An increase in kinetic energy is an increase in momentum that leads to a smaller wavelength. A smaller wavelength gives a smaller angle θ (see part (b)).

39.90. **IDENTIFY:** The photon is emitted as the atom returns to the lower energy state. The duration of the excited state limits the energy of that state due to the uncertainty principle.

SET UP: The wavelength λ of the photon is related to the transition energy E of the atom by $E = \dfrac{hc}{\lambda}$.

$\Delta E\,\Delta t \ge \hbar/2$. The minimum uncertainty in energy is $\Delta E \ge \dfrac{\hbar}{2\Delta t}$.

EXECUTE: **(a)** The photon energy equals the transition energy of the atom, 3.50 eV.

$$\lambda = \frac{hc}{E} = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{3.50\ \text{eV}} = 355\ \text{nm}.$$

(b) $\Delta E = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(4.0\times10^{-6}\ \text{s})} = 1.32\times10^{-29}\ \text{J} = 8.2\times10^{-11}\ \text{eV}.$

EVALUATE: The uncertainty in the energy could be larger than that found in (b), but never smaller.

39.91. **IDENTIFY:** The wave (light or electron matter wave) having less energy will cause less damage to the virus.

SET UP: For a photon $E_{\text{ph}} = \dfrac{hc}{\lambda} = \dfrac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{\lambda}$. For an electron $E_{\text{e}} = \dfrac{p^2}{2m} = \dfrac{h^2}{2m\lambda^2}$.

EXECUTE: **(a)** $E = \dfrac{hc}{\lambda} = \dfrac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{5.00\times10^{-9}\ \text{m}} = 248\ \text{eV}.$

(b) $E_{\text{e}} = \dfrac{h^2}{2m\lambda^2} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})(5.00\times10^{-9}\ \text{m})^2} = 9.65\times10^{-21}\ \text{J} = 0.0603\ \text{eV}.$

EVALUATE: The electron has much less energy than a photon of the same wavelength and therefore would cause much less damage to the virus.

39.92. **IDENTIFY** and **SET UP:** Assume $px \approx h$ and use this to express E as a function of x. E is a minimum for that x that satisfies $\dfrac{dE}{dx} = 0.$

EXECUTE: **(a)** Using the given approximation, $E = \dfrac{1}{2}((h/x)^2/m + kx^2), (dE/dx) = kx - (h^2/mx^3),$ and the minimum energy occurs when $kx = (h^2/mx^3)$, or $x^2 = \dfrac{h}{\sqrt{mk}}$. The minimum energy is then $h\sqrt{k/m}$.

EVALUATE: **(b)** $U = \tfrac{1}{2}kx^2 = \dfrac{h}{2}\sqrt{\dfrac{k}{m}}$. $K = \dfrac{p^2}{2m} = \dfrac{h^2}{2mx^2} = \dfrac{h}{2}\sqrt{\dfrac{k}{m}}$. At this x the kinetic and potential energies are the same.

39.93. **(a) IDENTIFY** and **SET UP:** $U = A|x|$. Eq. (7.17) relates force and potential. The slope of the function $A|x|$ is not continuous at $x = 0$ so we must consider the regions $x > 0$ and $x < 0$ separately.

EXECUTE: For $x > 0, |x| = x$ so $U = Ax$ and $F = -\dfrac{d(Ax)}{dx} = -A.$ For $x < 0$, $|x| = -x$ so $U = -Ax$ and $F = -\dfrac{d(-Ax)}{dx} = +A.$ We can write this result as $F = -A|x|/x$, valid for all x except for $x = 0$.

(b) IDENTIFY and **SET UP:** Use the uncertainty principle, expressed as $\Delta p \Delta x \approx h$, and as in Problem 39.80 estimate Δp by p and Δx by x. Use this to write the energy E of the particle as a function of x. Find the value of x that gives the minimum E and then find the minimum E.

EXECUTE: $E = K + U = \dfrac{p^2}{2m} + A|x|$

$px \approx h$, so $p \approx h/x$

Then $E \approx \dfrac{h^2}{2mx^2} + A|x|.$

For $x > 0, E = \dfrac{h^2}{2mx^2} + Ax.$

To find the value of x that gives minimum E set $\dfrac{dE}{dx} = 0.$

$$0 = \frac{-2h^2}{2mx^3} + A$$

$$x^3 = \frac{h^2}{mA} \text{ and } x = \left(\frac{h^2}{mA}\right)^{1/3}$$

With this x the minimum E is

$$E = \frac{h^2}{2m}\left(\frac{mA}{h^2}\right)^{2/3} + A\left(\frac{h^2}{mA}\right)^{1/3} = \frac{1}{2}h^{2/3}m^{-1/3}A^{2/3} + h^{2/3}m^{-1/3}A^{2/3}$$

$$E = \frac{3}{2}\left(\frac{h^2A^2}{m}\right)^{1/3}$$

EVALUATE: The potential well is shaped like a V. The larger A is, the steeper the slope of U and the smaller the region to which the particle is confined and the greater is its energy. Note that for the x that minimizes E, $2K = U$.

39.94. **(a) IDENTIFY and SET UP:** Let the y-direction be from the thrower to the catcher, and let the x-direction be horizontal and perpendicular to the y-direction. A cube with volume $V = 125\text{ cm}^3 = 0.125 \times 10^{-3}\text{ m}^3$ has side length $l = V^{1/3} = (0.125 \times 10^{-3}\text{ m}^3)^{1/3} = 0.050\text{ m}$. Thus estimate Δx as $\Delta x \approx 0.050\text{ m}$. Use the uncertainty principle to estimate Δp_x.

EXECUTE: $\Delta x \Delta p_x \ge \hbar/2$ then gives $\Delta p_x \approx \dfrac{\hbar}{2\Delta x} = \dfrac{0.01055\text{ J}\cdot\text{s}}{2(0.050\text{ m})} = 0.11\text{ kg}\cdot\text{m/s}$. (The value of $\hbar$ in this other universe has been used.)

(b) IDENTIFY and SET UP: $\Delta x = (\Delta v_x)t$ is the uncertainty in the x-coordinate of the ball when it reaches the catcher, where t is the time it takes the ball to reach the second student. Obtain Δv_x from Δp_x.

EXECUTE: The uncertainty in the ball's horizontal velocity is $\Delta v_x = \dfrac{\Delta p_x}{m} = \dfrac{0.11\text{ kg}\cdot\text{m/s}}{0.25\text{ kg}} = 0.42\text{ m/s}$.

The time it takes the ball to travel to the second student is $t = \dfrac{12\text{ m}}{6.0\text{ m/s}} = 2.0\text{ s}$. The uncertainty in the x-coordinate of the ball when it reaches the second student that is introduced by Δv_x is $\Delta x = (\Delta v_x)t = (0.42\text{ m/s})(2.0\text{ s}) = 0.84\text{ m}$. The ball could miss the second student by about 0.84 m.

EVALUATE: A game of catch would be very different in this universe. We don't notice the effects of the uncertainty principle in everyday life because h is so small.

39.95. **IDENTIFY and SET UP:** The period was found in Exercise 39.29b: $T = \dfrac{4\epsilon_0^2 n^3 h^3}{me^4}$. Eq. (39.14) gives the energy of state n of a hydrogen atom.

EXECUTE: **(a)** The frequency is $f = \dfrac{1}{T} = \dfrac{me^4}{4\epsilon_0^2 n^3 h^3}$.

(b) Eq. (39.5) tells us that $f = \dfrac{1}{h}(E_2 - E_1)$. So $f = \dfrac{me^4}{8\epsilon_0^2 h^3}\left(\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}\right)$ (from Eq. (39.14)). If

$n_2 = n$ and $n_1 = n+1$, then $\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2} = \dfrac{1}{n^2} - \dfrac{1}{(n+1)^2} = \dfrac{1}{n^2}\left(1 - \dfrac{1}{(1+1/n)^2}\right) \approx \dfrac{1}{n^2}\left(1 - \left(1 - \dfrac{2}{n} + \ldots\right)\right) = \dfrac{2}{n^3}$.

Therefore, for large n, $f \approx \dfrac{me^4}{4\epsilon_0^2 n^3 h^3}$.

EVALUATE: We have shown that for large n we obtain the classical result that the frequency of revolution of the electron is equal to the frequency of the radiation it emits.

39.96. **IDENTIFY:** Follow the steps specified in the hint.

SET UP: The value of Δx_i that minimizes Δx_f satisfies $\dfrac{d(\Delta x_f)}{d(\Delta x_i)} = 0.$

EXECUTE: Time of flight of the marble, from a free-fall kinematic equation is just $t = \sqrt{\dfrac{2y}{g}} = \sqrt{\dfrac{2(25.0\text{ m})}{9.80\text{ m/s}^2}} = 2.26\text{ s}.$ $\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \left(\dfrac{\Delta p_x}{m}\right)t = \dfrac{\hbar t}{2\Delta x_i m} + \Delta x_i.$ To minimize Δx_f with respect to Δx_i, $\dfrac{d(\Delta x_f)}{d(\Delta x_i)} = 0 = \dfrac{-\hbar t}{2m(\Delta x_i)^2} + 1 \Rightarrow \Delta x_i(\text{min}) = \sqrt{\left(\dfrac{\hbar t}{2m}\right)}$

$$\Rightarrow \Delta x_f(\text{min}) = \sqrt{\frac{\hbar t}{2m}} + \sqrt{\frac{\hbar t}{2m}} = \sqrt{\frac{2\hbar t}{m}} = \sqrt{\frac{2(1.055\times 10^{-34}\text{ J}\cdot\text{s})(2.26\text{ s})}{0.0200\text{ kg}}} = 1.54\times 10^{-16}\text{ m} = 1.54\times 10^{-7}\text{ nm}.$$

EVALUATE: The uncertainty introduced by the uncertainty principle is completely negligible in this situation.